

## Distribution Ray Tracing

## Distribution Ray Tracing

use many rays to compute average values over pixel areas, time, area lights, reflected directions, ...

## Antialiasing Origin

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- compute average color subtended by a pixel

pixel center  $C = \text{raytrace}(\mathbf{E}, \mathbf{Q} - \mathbf{E})$

pixel average  $C = \frac{1}{A_p} \cdot \int_{\mathbf{Q} \in P} \text{raytrace}(\mathbf{E}, \mathbf{Q} - \mathbf{E}) \cdot dA_{\mathbf{Q}}$

$\mathbf{E}$ : camera origin

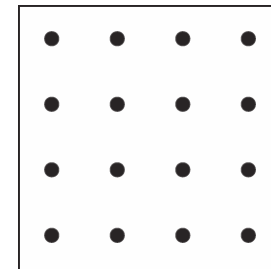
$\mathbf{Q}$ : point on image plane

$P$ : pixel of area  $A_p$

## Antialiasing by Deterministic Integration

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- subdivide the pixel in squares
- cast rays through squares centers
- average result

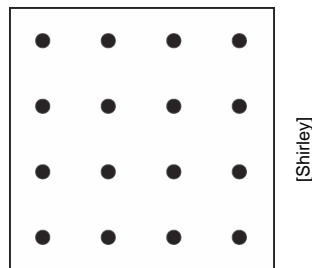


[Shirley]

## Deterministic Antialiasing Pseudocode

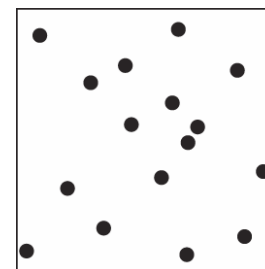
- antialiasing pixel (i, j)

```
c = 0
for sx = 0 to ns
  for sy = 0 to ns
    u = (i + (sx+0.5)/ns) / width
    v = (j + (sy+0.5)/ns) / height
    Q = imagePlanePoint(u,v)
    c += raytrace(E,Q-E)
c /= ns2
```



## Antialiasing by Monte Carlo Estimation

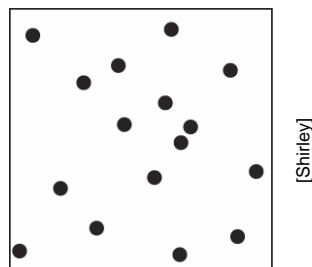
- pick random points in pixel area
- cast rays through them
- average result



## Monte Carlo Antialiasing Pseudocode

- antialiasing pixel (i, j)

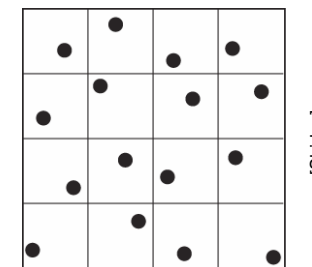
```
c = 0
for s = 0 to ns2
  (rx,ry) = random2d();
  u = (i + rx) / width
  v = (j + ry) / height
  Q = imagePoint(u,v)
  c += raytrace(E,Q-E)
c /= ns2
```



## Monte Carlo Antialiasing Pseudocode

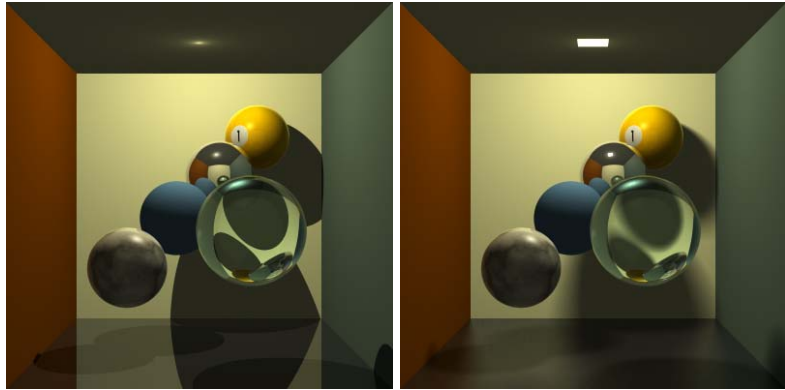
- antialiasing pixel (i, j)

```
c = 0
for sx = 0 to ns
  for sy = 0 to ns
    (rx,ry) = random2d();
    u = (i + (sx+rx)/ns) / width
    v = (j + (sy+ry)/ns) / height
    Q = imagePoint(u,v)
    c += raytrace(E,Q-E)
c /= ns2
```



## Raytraced Images are too "Clean"

- soft shadows come from area light
  - raytracing only supports point lights



[Jason Waltman / jasonwaltman.com]

## Raytraced Images are too "Clean"

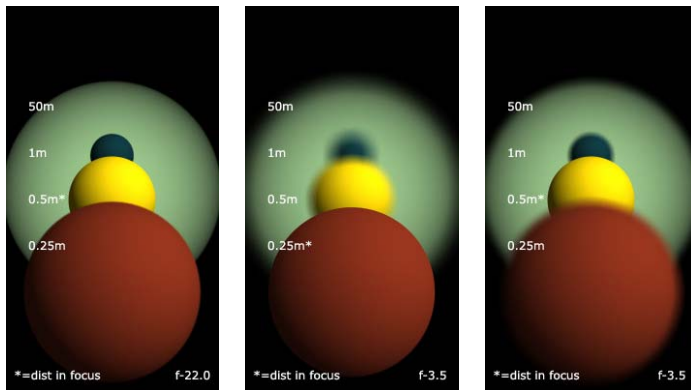
- blurry reflections come from rough materials
  - raytracing only supports perfectly sharp mirrors



[Jensen]

## Raytraced Images are too "Clean"

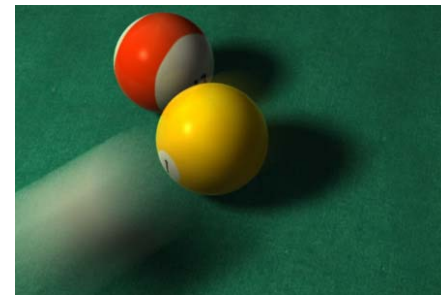
- depth of field come from lens system
  - raytracing only supports pinhole camera



[Jason Waltman / jasonwaltman.com]

## Raytracer Images are too "Clean"

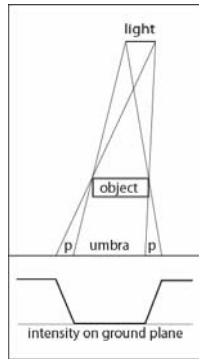
- motion blur come from shutter speed
  - raytracing only support infinitely fast shutter speed



[Jason Waltman / jasonwaltman.com]

## Soft Shadows Origin

- area lights create penumbras
  - light is only partially visible from a given point
  - want to compute how much light hits the point



[Shirley]

## Approximate Soft Shadows Principle

point light  $C = C_l \cdot V(\mathbf{P}, \mathbf{S}) \cdot \text{shading}(\mathbf{P}, \mathbf{S})$

area light  $C = \frac{C_l}{A_L} \cdot \int_{S \in L} V(\mathbf{P}, \mathbf{S}) \cdot \text{shading}(\mathbf{P}, \mathbf{S}) \cdot dA_S$

$\mathbf{P}$  : point on the surface

$\mathbf{S}$  : point on the light

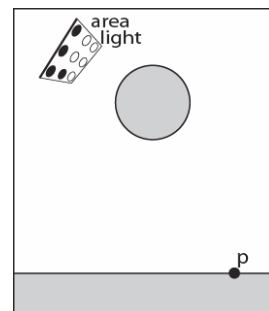
$V$  : visibility function (0 or 1)

$L$  : light of area  $A_L$

$C_l$  : total light intensity

## Soft Shadows by Deterministic Integration

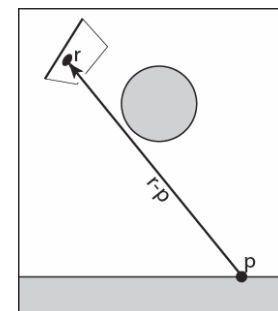
- approximate area light as a set of point lights
  - equivalent to quadrature rule
- for each point, compute shadows and lighting
- average results



[Shirley]

## Soft Shadows by Monte Carlo Estimation

- use Monte Carlo integration
- pick random points on the light
  - easy for quad lights, hard (but possible) for others
- compute shadows and lighting
- average results



[Shirley]

## Soft Shadows by Monte Carlo Estimation

$$\mathbf{S}_i = \mathbf{S}_c + (0.5 - r_{i,1})l\mathbf{u} + (0.5 - r_{i,2})l\mathbf{v}$$

for quads

$$\langle C \rangle = \frac{C_l}{N} \cdot \sum_i^N V(\mathbf{P}, \mathbf{S}_i) \cdot \text{shading}(\mathbf{P}, \mathbf{S}_i)$$

$\mathbf{S}_c$  : light source center

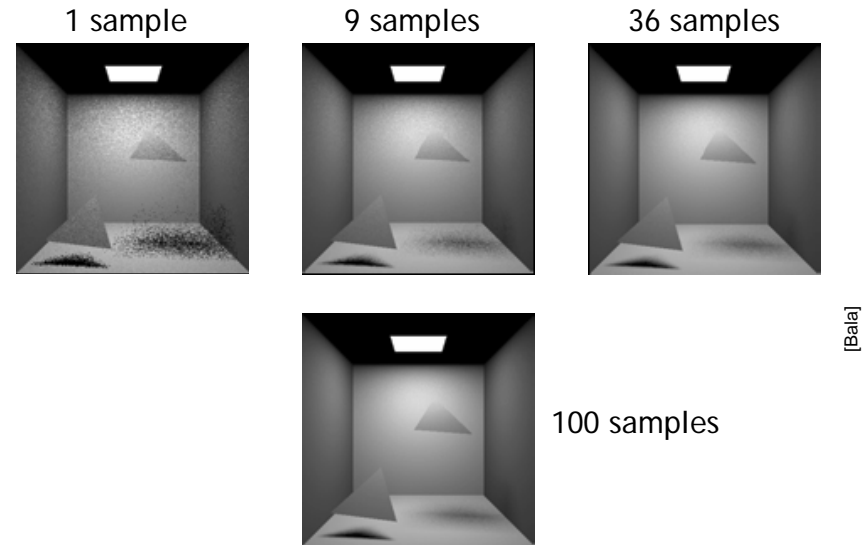
$\mathbf{u}, \mathbf{v}$  : light source tangent vectors

$l$  : light source size

$r_i$  : uniformly sampled random 2d vector in  $[0, 1]^2$

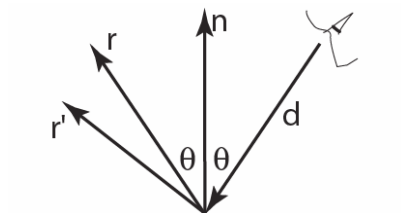
$N$  : total number of samples

## How Many Samples?



## Blurry Reflections Origin

- real materials often have blurred reflections
  - light is scattered around a set of directions
  - want to compute how much light reaches the surface along these directions



[Shirley]

## Approximate Blurry Reflections Principle

mirror refl.       $C = k_r \cdot \text{raytrace}(\mathbf{P}, \mathbf{R})$

blurry refl.       $C = \frac{k_r}{A_\Omega} \cdot \int_{\mathbf{R} \in \Omega} \text{raytrace}(\mathbf{P}, \mathbf{R}) \cdot dA_{\mathbf{R}}$

$\mathbf{P}$  : point on the surface

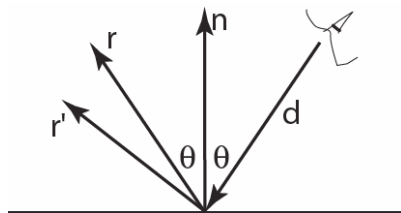
$\mathbf{R}$  : reflected direction

$\Omega$  : solid angle where object is reflecting

$k_r$  : reflection coefficient

## Blurred Reflection by Monte Carlo Est.

- compute reflected direction  $\mathbf{R}$
- pick random direction  $\mathbf{R}'$  around  $\mathbf{R}$
- compute reflected color along  $\mathbf{R}'$
- average results



[Shirley]

## Blurred Reflection by Monte Carlo Est.

$$\mathbf{R}_i = [\mathbf{R} + (0.5 - r_{i,1})l\mathbf{u} + (0.5 - r_{i,2})l\mathbf{v}] / |\mathbf{R}_i|$$

for quads

$$\langle C \rangle = \frac{k_r}{N} \cdot \sum_i^N \text{raytrace}(\mathbf{P}, \mathbf{R}_i)$$

$\mathbf{R}$ : reflected direction

$\mathbf{u}, \mathbf{v}$ : vectors orthogonal to  $\mathbf{R}$

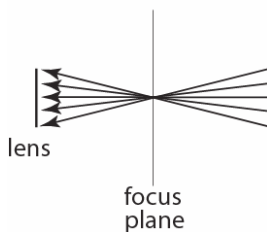
$l$ : blur size

$r_i$ : uniformly sampled random 2d vector in  $[0, 1]^2$

$N$ : total number of samples

## Depth-of-Field Origin

- images in eyes and cameras are formed by lenses
  - not all rays converge to a point
  - so only some object appear sharp (in focus)
  - while all other objects do not (out of focus)



[Shirley]

## Approximate Depth-of-Field Principle

pinhole  $C = \frac{1}{A_p} \int_{\mathbf{Q} \in A_p} \text{raytrace}(\mathbf{E}, \mathbf{Q} - \mathbf{E}) dA_{\mathbf{Q}}$

dof  $C = \frac{1}{A_p A_L} \cdot \int_{\mathbf{Q} \in P} \int_{\mathbf{F} \in L} \text{raytrace}(\mathbf{F}, \mathbf{Q} - \mathbf{F}) \cdot dA_{\mathbf{F}} \cdot dA_{\mathbf{Q}}$

$\mathbf{E}$ : camera origin

$\mathbf{Q}$ : point on image plane

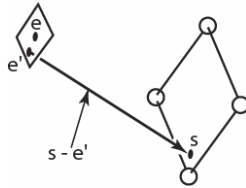
$\mathbf{F}$ : point on "lens" plane around  $\mathbf{E}$

$P$ : pixel of area  $A_p$

$L$ : lens of area  $A_L$

## Depth-of-Field by Monte Carlo Estimation

- pick random points on film and image plane
- compute color from aperture point toward image one
- average results



[Shirley]

## Depth-of-Field by Monte Carlo Estimation

$$\mathbf{F}_i = \mathbf{E} + (0.5 - s_{i,1})l_a \mathbf{u} + (0.5 - s_{i,2})l_a \mathbf{v}$$

for quads  $\mathbf{Q}_i = \mathbf{E} + (x + 0.5 - r_{i,1})l_p \mathbf{u} + (y + 0.5 - r_{i,2})l_p \mathbf{v} - n\mathbf{z}$

$$\langle C \rangle = \frac{1}{N} \cdot \sum_i^N \text{raytrace}(\mathbf{F}_i, \mathbf{Q}_i - \mathbf{F}_i)$$

$\mathbf{u}, \mathbf{v}$  : vectors parallel to image plane

$l_p, l_L$  : pixel/aperture size

$n$  : distance to image plane

$\mathbf{r}_i, \mathbf{s}_i$  : uniformly sampled random 2d vectors in  $[0, 1]^2$

$N$  : total number of samples

## Motion Blur Origin

- takes time for images to form on camera/eye
  - during that time the camera/eye is open
  - sensors take the "average" of what's happening
  - but objects move around in that time

## Approximate Motion Blur Principle

no blur  $C(t) = \text{raytrace}(\mathbf{E}, \mathbf{I}, S_t)$

blurry refl.  $C(t, \Delta t) = \frac{1}{\Delta t} \cdot \int_{s \in [t, t + \Delta t]} \text{raytrace}(\mathbf{E}, \mathbf{I}, S_s) \cdot dl_s$

$\mathbf{E}$  : any ray origin

$\mathbf{I}$  : any ray direction

$S_t$  : scene at time  $t$

$t$  : time when shutter opens

$\Delta t$  : time that shutter stays open

$\Delta t = t_{k+1} - t_k$  : for animation, time between frames

## Motion Blur by Monte Carlo Estimation

- for any ray (camera, shadow, reflection), pick random time in the shutter interval
- update object transformations for picked time
- compute intersection
- average results

## Motion Blur by Monte Carlo Estimation

for aliased camera rays  $\langle C \rangle = \frac{1}{N} \cdot \sum_i^N \text{raytrace}(\mathbf{E}, \mathbf{Q} - \mathbf{E}, S_{t_i})$

$\mathbf{E}$  : camera origin

$\mathbf{Q}$  : point on image plane

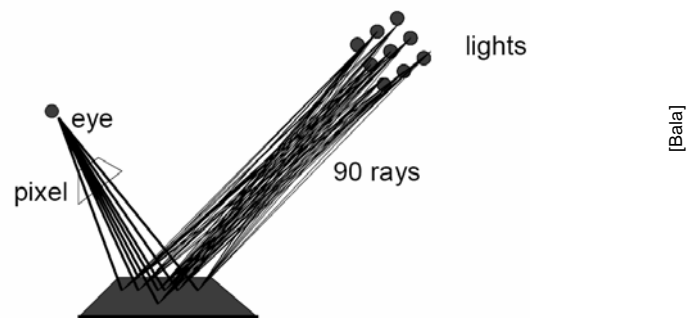
$t_i$  : uniformly sampled random variable in  $[0, 1]$

$N$  : total number of samples

## Combining Estimations

- e.g. compute soft shadows and antialias the pixel

$$C = \frac{C_l}{A_p A_L} \cdot \int_{\mathbf{Q} \in A_p} \int_{\mathbf{S} \in A_L} V(\mathbf{P}(\mathbf{Q}), \mathbf{S}) \cdot \text{shading}(\mathbf{P}(\mathbf{Q}), \mathbf{S}) \cdot dA_S \cdot dA_Q$$



## Combining Estimations: Path Tracing

- idea: *randomize the whole path*
  - compute all the integrals at once!

