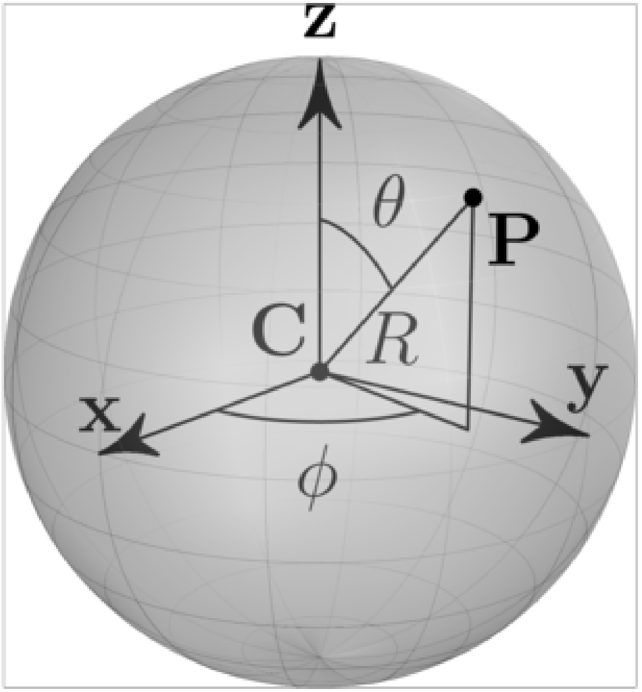
parametric surface patches

implicit representation

- implicit surface representation

$$f(\mathbf{P}) = 0$$

- e.g. sphere: $f(\mathbf{P}) = 0 \rightarrow x^2 + y^2 + z^2 - r^2 = 0$

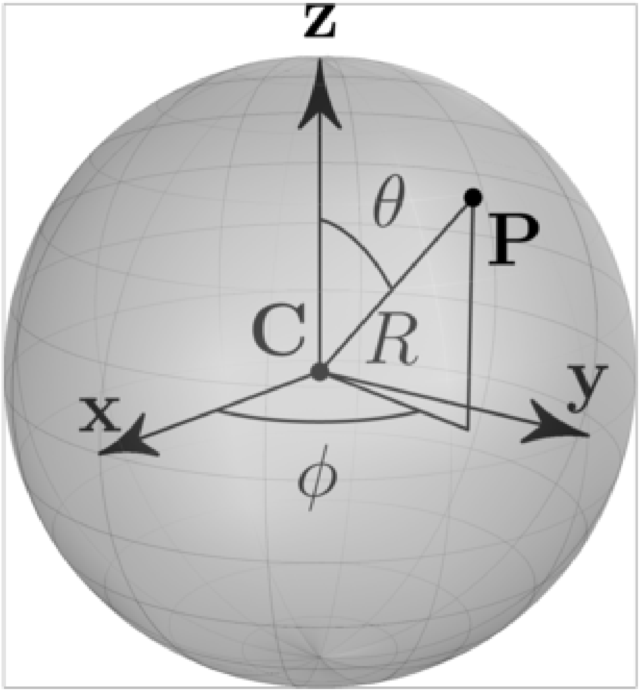


parametric representation

- parametric surface representation

$$\mathbf{P}(u) = (f_x(u), f_y(u), f_z(u))$$

- e.g. sphere: $\mathbf{P}(u) = (r\cos\phi\sin\theta, r\sin\phi\sin\theta, r\cos\theta)$



parametric representation

- goals when defining f
  - smoothness, efficiency, local control
  - same as curves
- (1) combine curves to obtain surfaces
  - easy for simple solids, but not general
- (2) extend parametric curves to surface patches
  - general formulation
  - used heavily in CAD

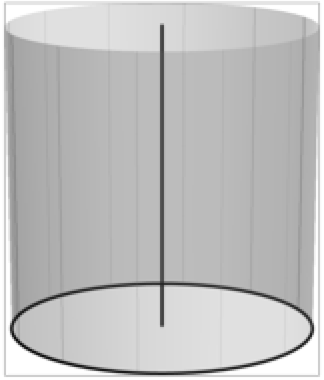
surfaces from curves

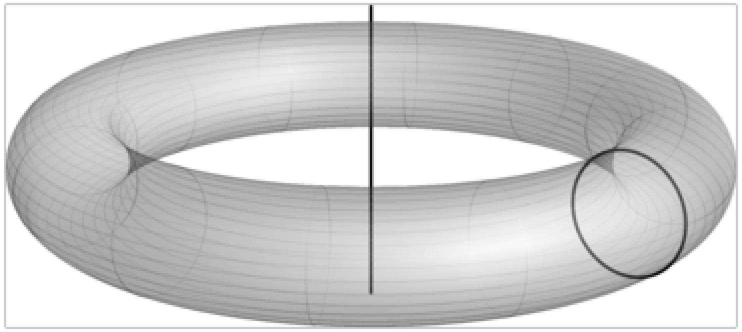
- example: extrude curve $\mathbf{C}(u)$ in XY-plane along Z-axis

$$\mathbf{P}(u,v) = \big(C_x(u), C_y(u), v\big)$$

- example: revolve curve $\mathbf{C}(u)$ in YZ-plane about Z-axis

$$\mathbf{P}(u,v) = \big(C_y(u)\cos(v), C_y(u)\sin(v), C_z(u)\big)$$





patches

patches

- spline curves: 1D blending functions

$$\mathbf{P}(t) = \sum_i b_i(t)\mathbf{P}_i$$

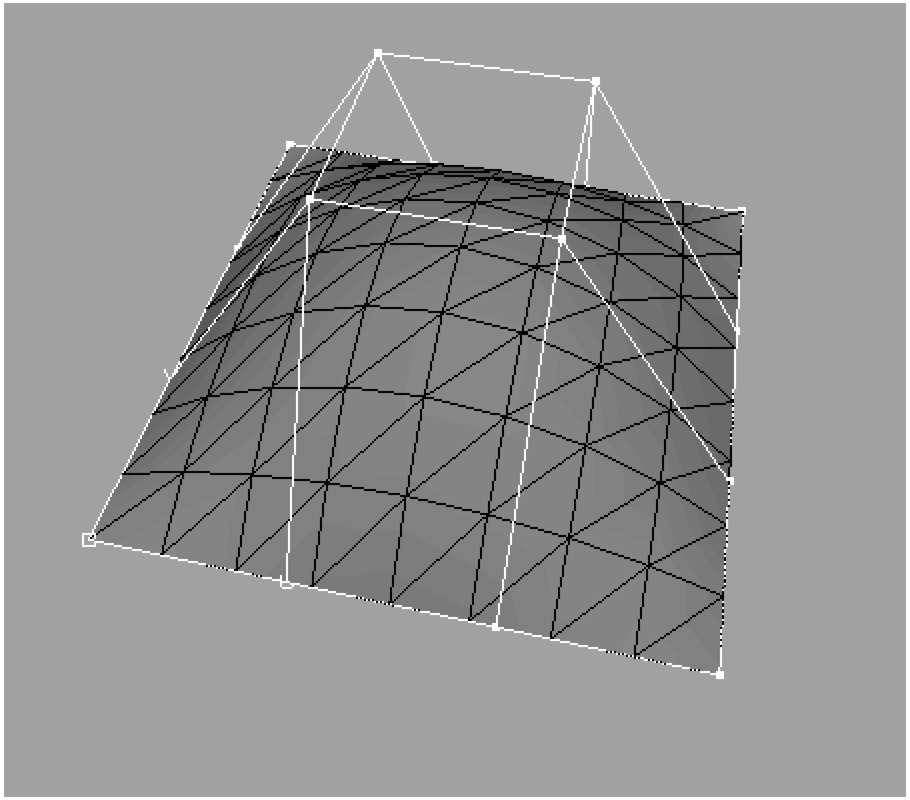
- surface patches: 2D blending functions
  - cross product of 1D blending functions

$$\mathbf{P}(u,v) = \sum_{ij} b_{ij}(u,v)\mathbf{P}_{ij}$$

$$b_{ij}(u,v) = b_i(u)b_j(v)$$

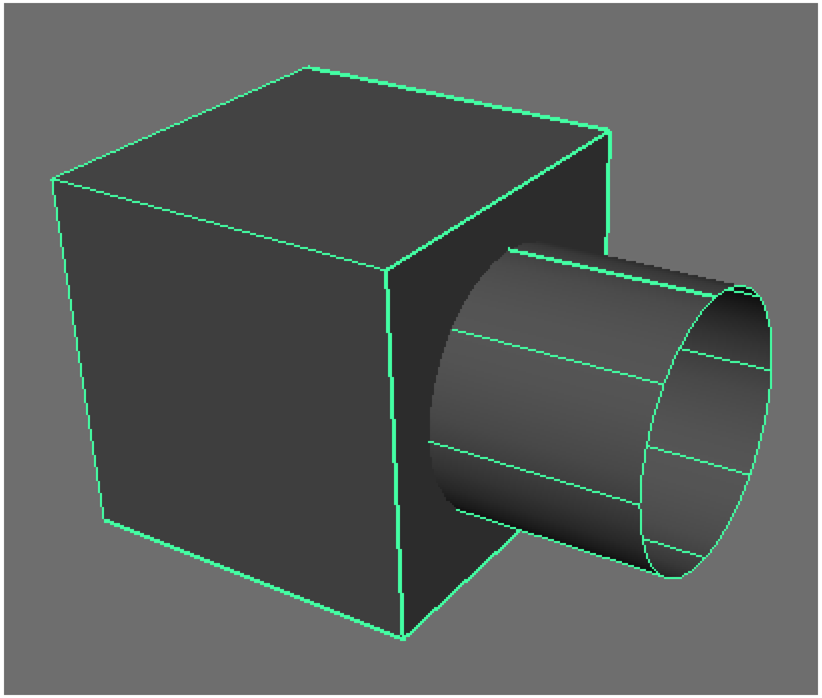
patches

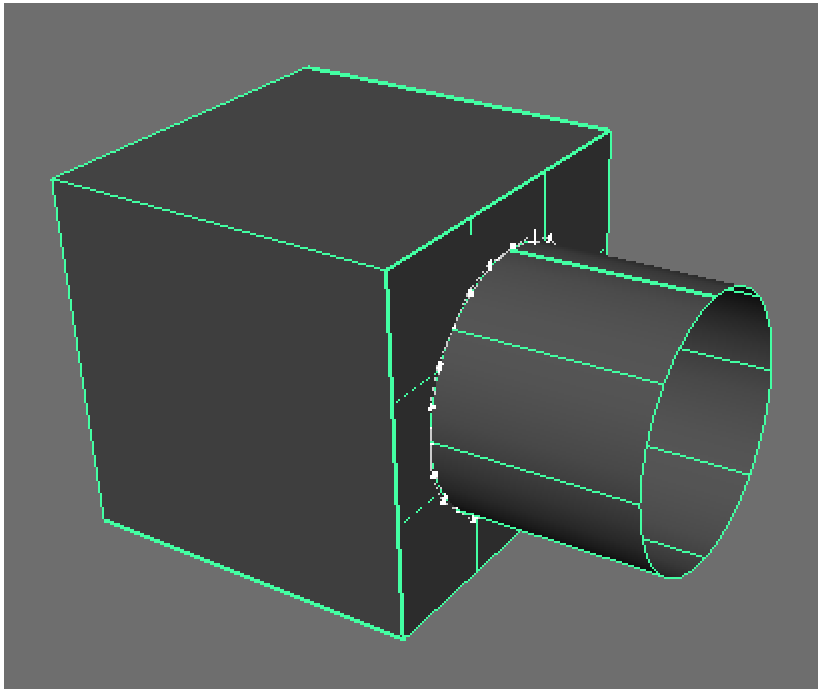
- bicubic Bezier patches



patches

- joining is hard





other patch functions

- just like curves
  - uniform, non-uniform
- non-uniform rational B-splines (NURBS)
  - ratios of B-splines
  - invariance under perspective
  - can represent conic sections exactly
  - often used in 3D

rendering parametric surfaces

rendering parametric surfaces

- tesselation: approximate surfaces with triangles/quads
  - meshes are efficient to draw in hardware and software
  - more faces to provide better approximation
- uniform tesselation: split (u, v) intervals uniformly
  - fast to compute and simple to implement
  - generates many segments
- adaptive tesselation: split recursively until good enough
  - Bezier patches can use De Castaljeu

uniform tesselation

- split in $K \times K$ points uniformly at $(u_{k_1}, v_{k_2}) = (1/k_1, 1/k_2)$
  - tesselate into quads by creating a $K \times K$ quad grid
  - set vertices to $\mathbf{P}(u_{k_1}, v_{k_2})$
  - join vertices to avoid holes if function wraps (e.g. cylinder)
- normals
  - evaluate analytically if possible (e.g. sphere)
  - or evaluate by computing partial derivatives $\mathbf{n} = \partial\mathbf{P}/\partial u \times \partial\mathbf{P}/\partial v$
  - or smooth mesh by averaging face normals over vertices