CS52: Homework 5

Out: Nov 19. Due: Dec 1.

Problem 1: Monte Carlo Integration

Suppose you want to compute the volume of an empty glass but you have no measuring instrument. The only thing you can do is to fill in a bottle of volume V of a random volume of liquid between 0 and V. By emptying the bottle in the glass you can test if it spills or not (not know how much spilling there is).

(a) Derive a random procedure with which you can estimate the volume of the glass only using the bottle and the test for spilling.

(b) Prove that the expected value of the procedure you have derived matches the volume of the glass.

Problem 2: Estimating Direct Illumination

You want to compute an image of objects directly illuminated by the sun and the sky. You can model the sky as an infinetly distant hemisphere of constant low-intensity light. You can model the sun as a small high-intensity light disk oriented toward the normal of each point $\mathbf{N_x}$ and placed at a very long distance away from any object.

(a) Write an efficient Monte Carlo estimator you can use to compute the direct illumination coming from the sun-sky model. Be precise in writing the estimator, and explicit on what pdf you are using and how you are to generate the samples. Also describe why you think the chosen estimator is efficient.

(b) Will the shadows from the sun be fully black? Will they be sharp or blurry? If the object is non-convex, there will be "holes" on the surface. What will happen to the lighting in those areas and why?

(c) In an overcast day (sun not visible) you can clearly see some kind of darkening below cars on the road. What can this be attributed to?

Problem 3: Rendering Equation

(a) Write formally the rendering equation corresponding to the estimators used in Problem 2.a.

Problem 4: Path Tracing Computational Complexity

You are rendering a scene using a path tracing algorithm to generate an image of $w \times h$ pixels, each of which is computed using n_p samples. Your implementation estimates the direct illumination using the area formulation with n_d samples while the indirect illumination is sampled in the hemispherical domain with n_i samples.

(a) You have implemented a simple stopping criterion. The path tracer will stop recursing after l recursions. Compute the total number of rays needed for the image.

(b) You have implemented a russian roulette stopping criterion. At each intersection, the path tracer will randomly stop to recurse with a probability α. Compute the total number of rays needed for the image.

Extra credit: None

No extra credit this time. You should try to do the programming assignment extra credit instead.