CS52: Homework 2

Out: Oct 8. Due: Oct 22.

Problem 1: Raytracing Computational Complexity

The most expensive operation in a raytracer is casting a ray to determine visibility. In this exercise you will compute the number of rays required to render an image; if all rays cost roughly the same, this number will also give you an estimate of the computational complexity of the rendering process (note that for optimized raytracers shadow rays cost less than normal ones).

(a) Give the formula to compute the total number of rays and the total number of intersection tests required when rendering an image of w by h pixels, antialiased using s^2 samples per pixel, of a scene of o objects illuminated by l lights and where the fraction of rays that hit an object is p.

(b) Work out the number of intersection tests required for the following cases:

1. Simple scene (low quality): $w = 128, h = 128, s = 1, o = 5, l = 1, p = 0.75$
2. Simple scene (high quality): $w = 2048, h = 2048, s = 5, o = 5, l = 1, p = 0.75$
3. Complex scene (high quality): $w = 2048, h = 2048, s = 5, o = 10000, l = 10, p = 1$

Problem 2: Shading

You are photographing a scene with a camera and a point light placed in the same position (an approximation of the way flashes are mounted on the top of cameras). The scene contains only a plane parallel to the camera image plane and at a distance d from it. The plane is made of a Phong material of k_d diffuse coefficient, k_s specular coefficient and n specular exponent, while the light has $C_l = (1, 1, 1)$. For this problem, the light has no distance falloff.

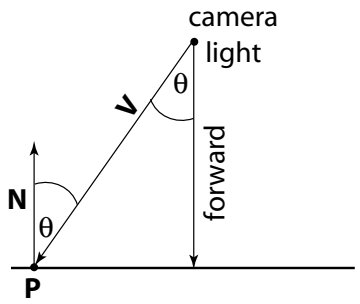
(a) Give the color $C(\theta)$ of a point on the plane viewed by a ray that forms an angle θ with the forward direction of the camera.

(b) Determine the angle θ for which $C(\theta) = 0.5C(0)$ in the following cases.

1. $k_d = (1.0, 1.0, 1.0), k_s = (0.0, 0.0, 0.0), n = 100$

2. $k_d = (0.0, 0.0, 0.0), k_s = (1.0, 1.0, 1.0), n = 100$

Based on these results, comment on why it is hard to use the flash on a shiny surface.



Problem 3: Lighting

In the setup of Problem 2, prove that there will be no point with a shadow visible in the image.

Problem 4: Hierarchical Transforms

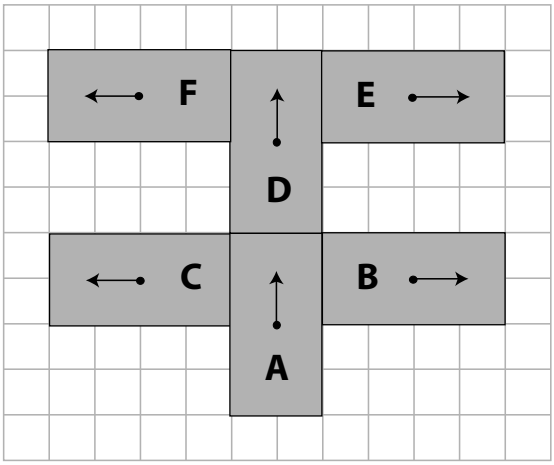
The construction in the following figure is made of six identical boxes of size (2,4) oriented as indicated by the arrows. Box A is centered at coordinates (0,0).

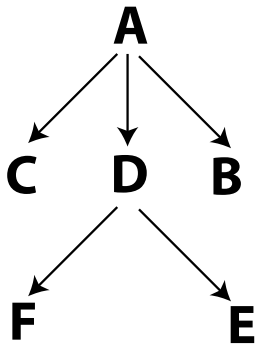
(a) Write the 3x3 transformation matrix that would transform A into

1. A $\rightarrow$ B
2. A $\rightarrow$ D
3. A $\rightarrow$ F

(b) An alternative way to describe the arrangement of objects in the picture is to define their transformation with respect to each other when arranged in a hierarchy shown below. Write the sequence of 3x3 matrices that would transform A into

1. A $\rightarrow \cdots \rightarrow$ B
2. A $\rightarrow \cdots \rightarrow$ D
3. A $\rightarrow \cdots \rightarrow$ F





Extra credit: Raytracing Computational Complexity

Extend the solution to Problem 1 for the case where a fraction r of rays that hit an object have mirror reflections. Given only the symbolic form solution, not the numerical ones.