subdivision surfaces

subdivision curves

- start with a piecewise linear curve
- apply recursively subdivision rule

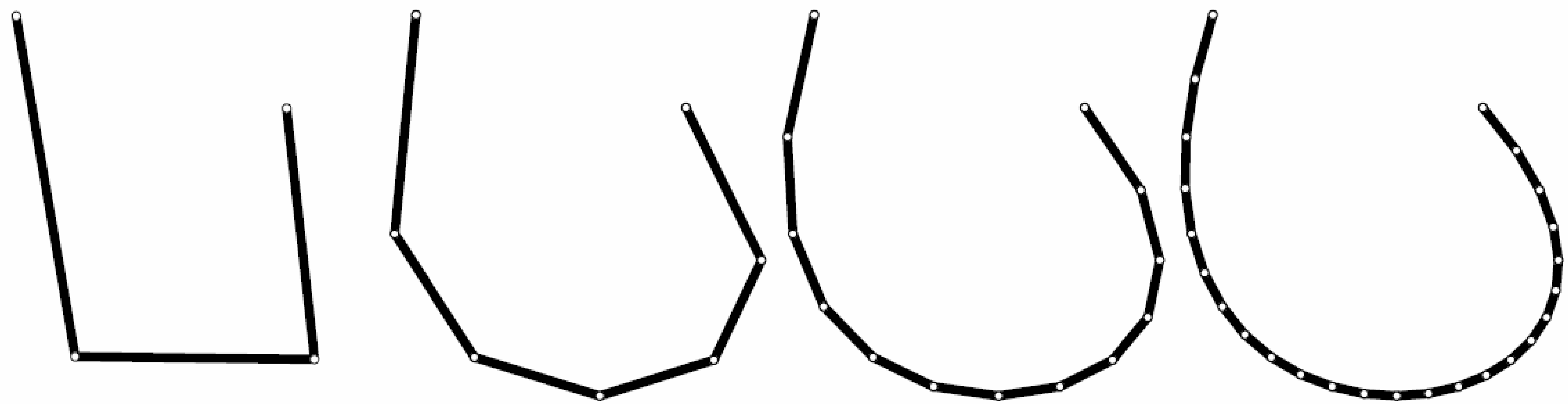


Figure 2.1: *Example of subdivision for curves in the plane. On the left 4 points connected with straight line segments. To the right of it a refined version: 3 new points have been inserted "inbetween" the old points and again a piecewise linear curve connecting them is drawn. After two more steps of subdivision the curve starts to become rather smooth.*

[Zorin and Schröder,2000]

subdivision surfaces

- start with polygon mesh
- apply recursively subdivision rule

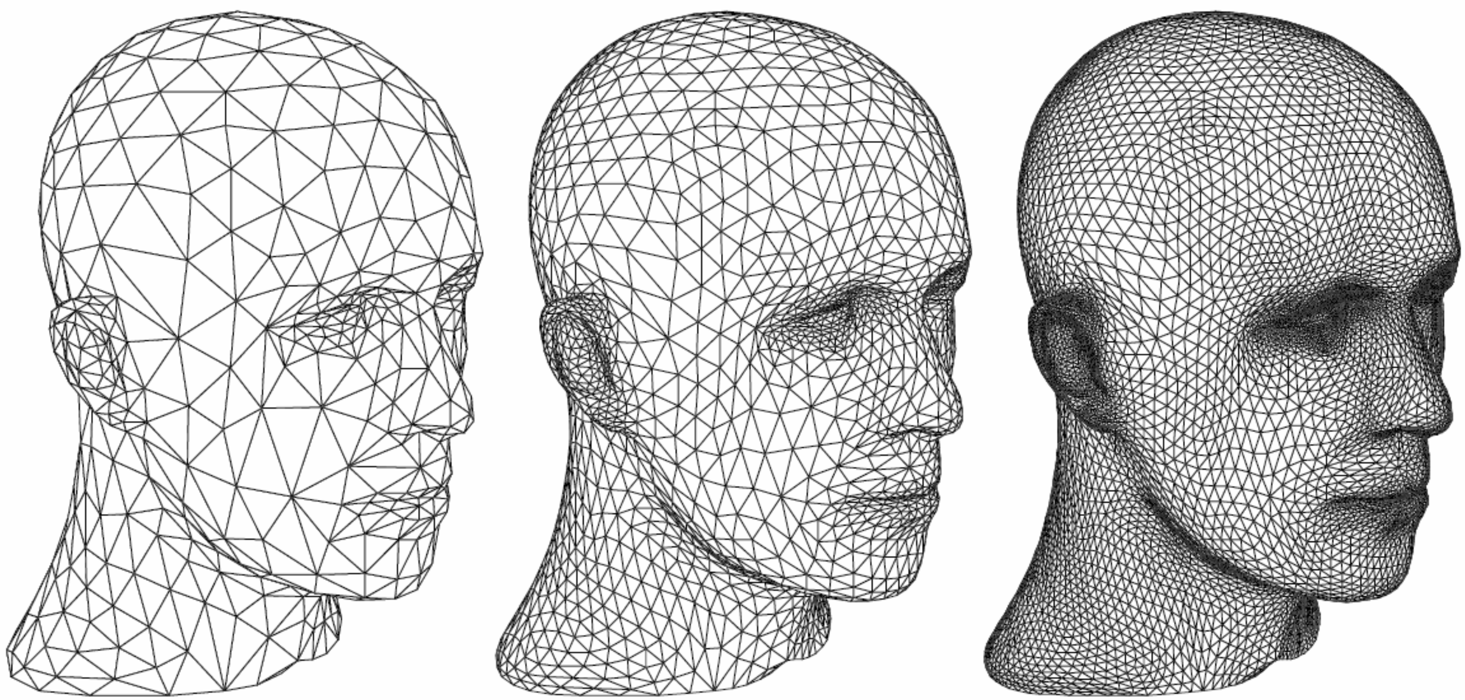


Figure 2.2: *Example of subdivision for a surface, showing 3 successive levels of refinement. On the left an initial triangular mesh approximating the surface. Each triangle is split into 4 according to a particular subdivision rule (middle). On the right the mesh is subdivided in this fashion once again.*

[Zorin and Schröder,2000]

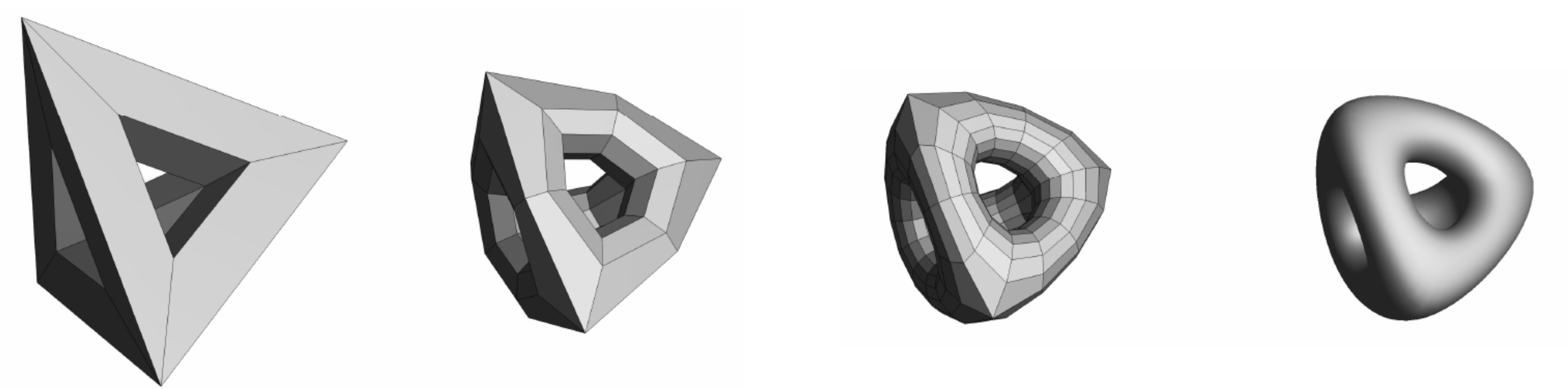
subdivision surfaces

- subdiv.: limit surface of subdivision process
  - provably smooth surface! (i.e. continuous function)
  - subdiv. process: input is mesh, output is refined mesh

$$\mathbf{M}_{i+1} = f(\mathbf{M}_i)$$

  - limit surface

$$\mathbf{S} = \lim_{i \to \infty} \mathbf{M}_i$$



[DeRose et al.,1998]

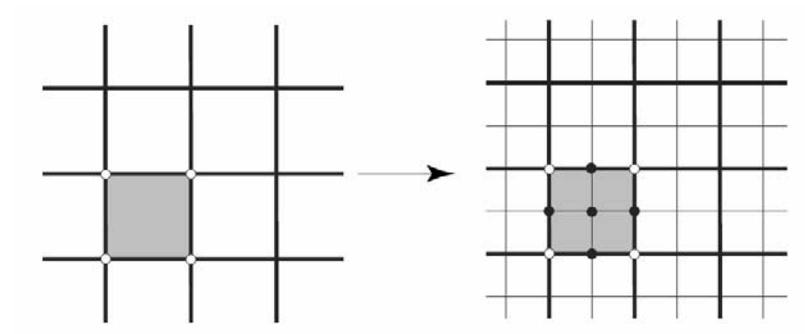
subdivision schemes

- two stages
- mesh subdivision → new topology
  - create new vertices and faces
- vertex placement → new geometry
  - compute vertex position

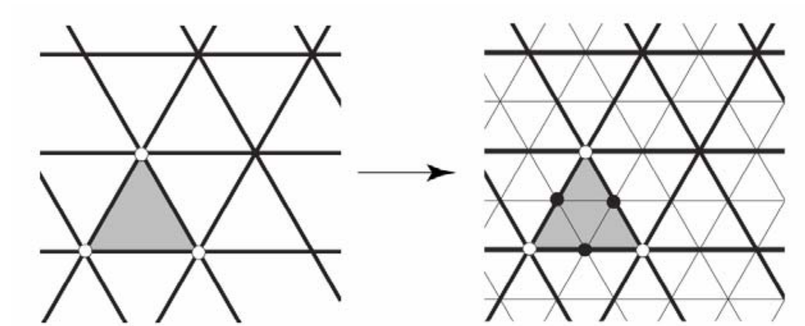
subdivision schemes

- different schemes available
  - depend on input/output topology and geometry

- quadrilaterals input:
  - Catmull-Clark

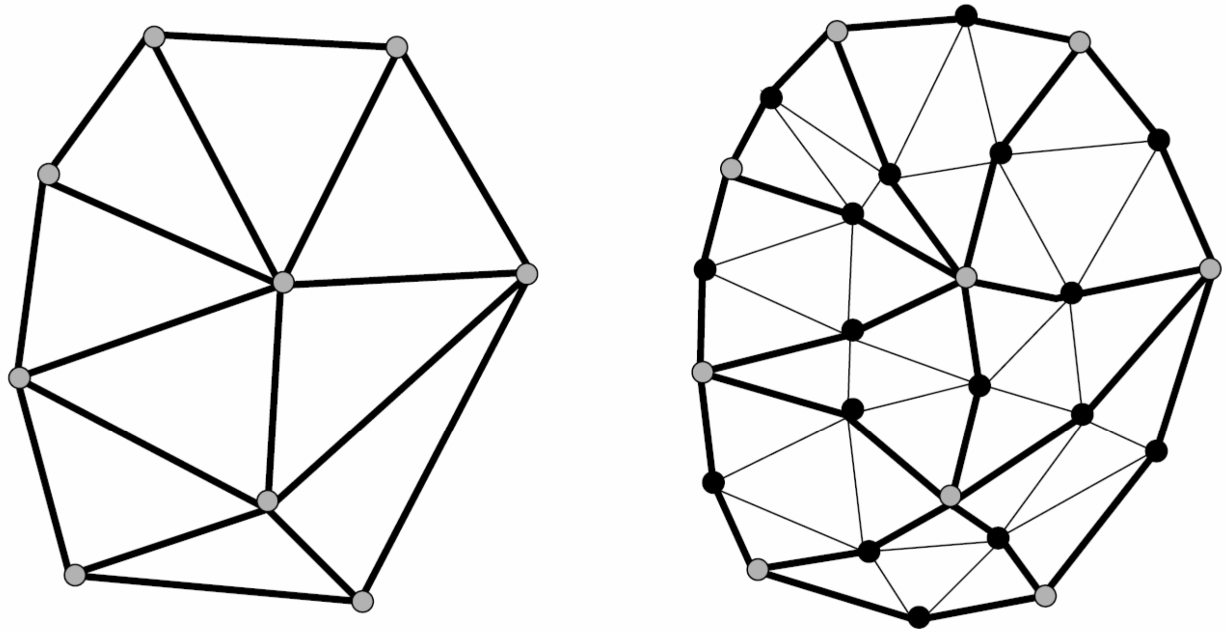


- triangles input:
  - Loop



loop subdivision scheme

- new topology: new faces
  - split each triangle in 4 triangles

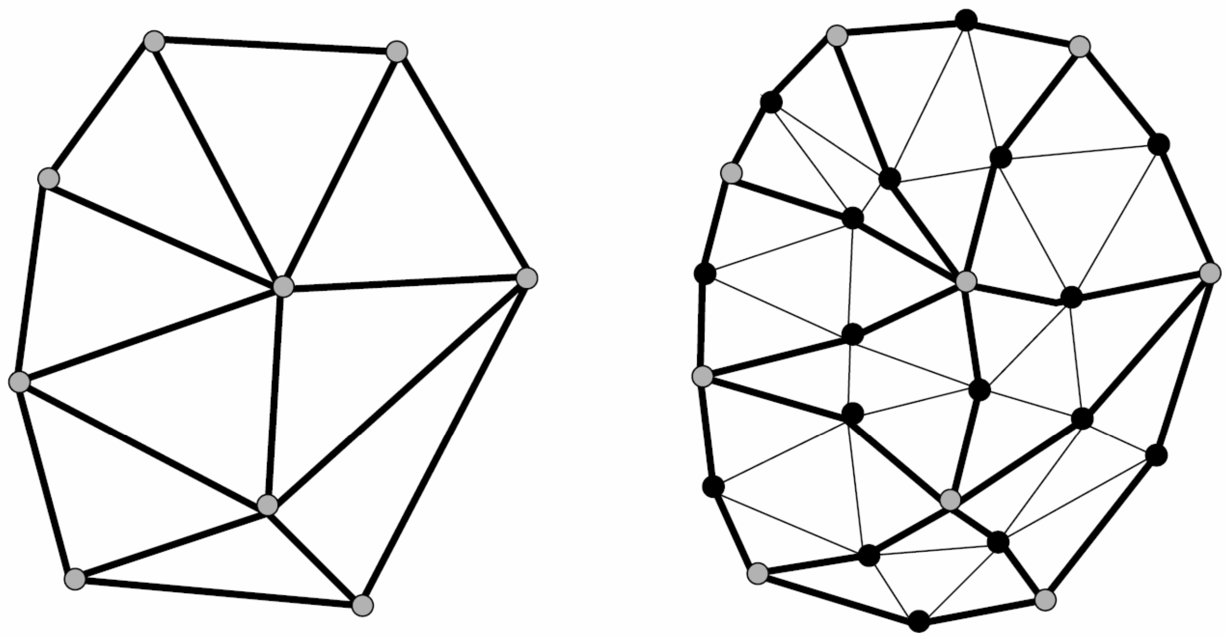


[Zorin and Schröder,2000]

Figure 3.1: *Refinement of a triangular mesh. New vertices are shown as black dots. Each edge of the control mesh is split into two, and new vertices are reconnected to form 4 new triangles, replacing each triangle of the mesh.*

loop subdivision scheme

- new geometry: new vertex positions
  - need rules for even/odd (white/black) vertices

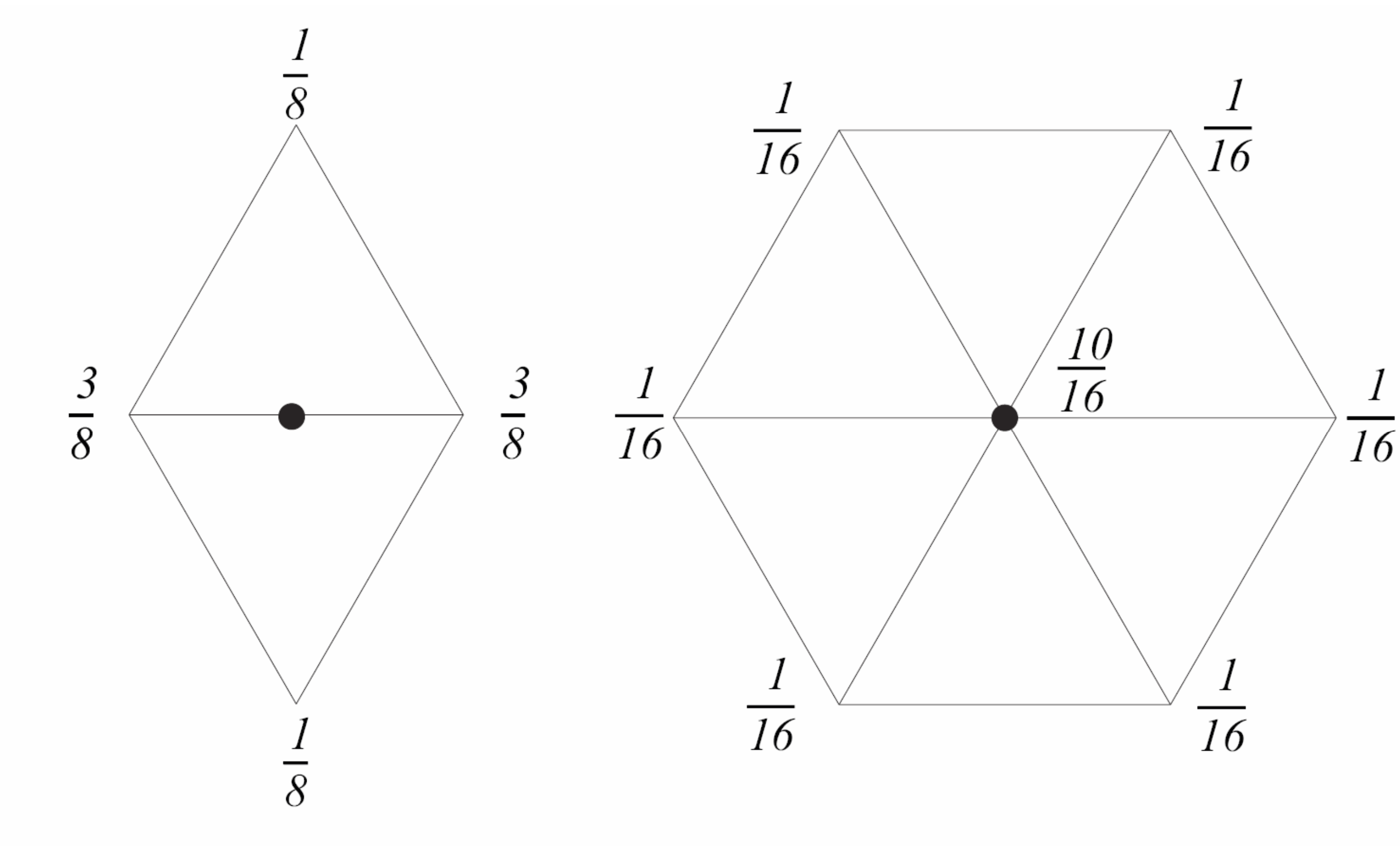


[Zorin and Schröder,2000]

Figure 3.1: *Refinement of a triangular mesh. New vertices are shown as black dots. Each edge of the control mesh is split into two, and new vertices are reconnected to form 4 new triangles, replacing each triangle of the mesh.*

loop subdivision rules

- new vertex pos: weighted average of neighbors



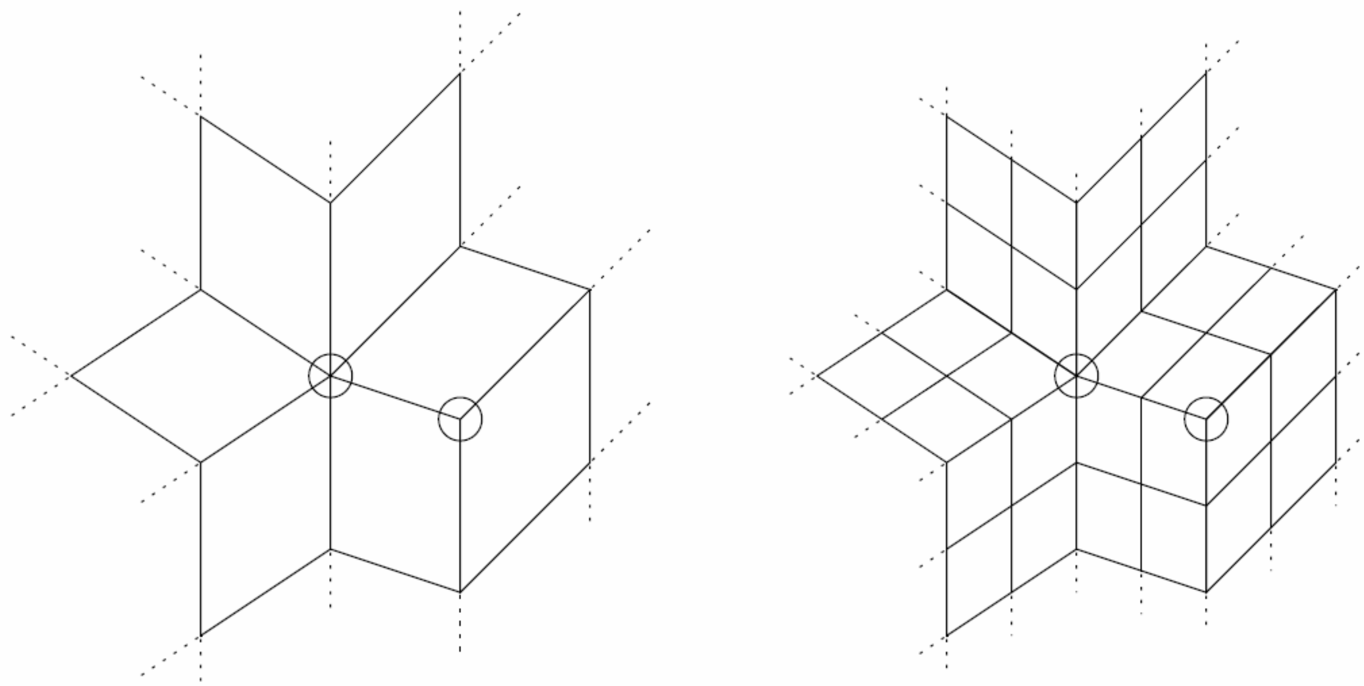
[Zorin and Schröder,2000]

boundaries and creases

- rules on boundaries
  - only consider vertices along the boundary
- creases: mark edges sharp
  - only consider vertices along a crease

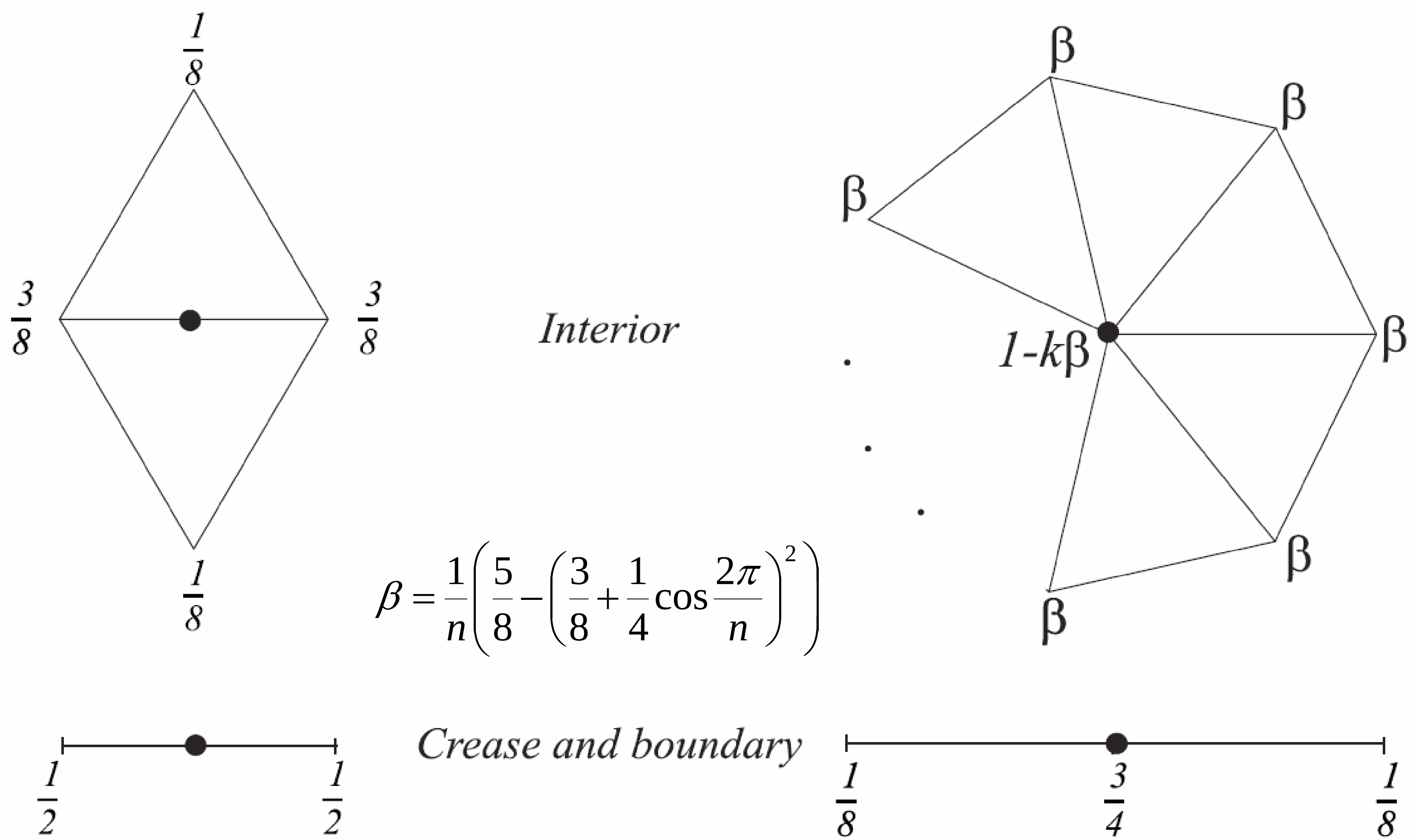
extraordinary vertices

- vertices with non-standard num. of neighbors
  - unavoidable for complex topology
  - make subdivision ideal for this case



[Zorin and Schröder,2000]

loop subdivision rules

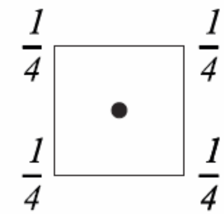


a. Masks for odd vertices

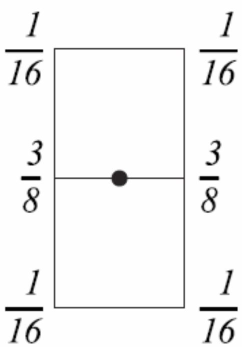
b. Masks for even vertices

[Zorin and Schröder,2000]

Catmull-Clark subdivision rules

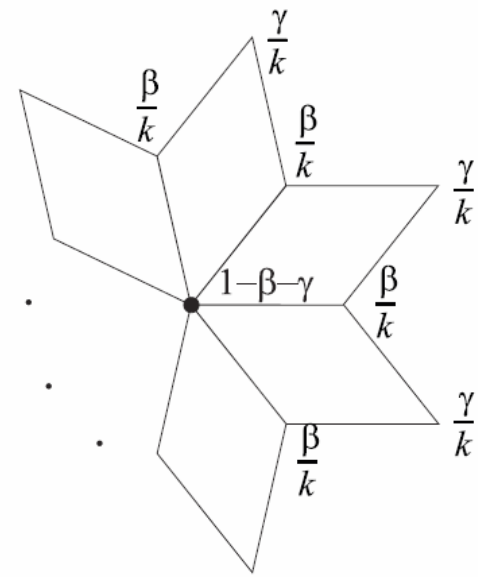


Mask for a face vertex



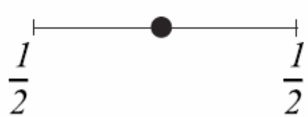
Mask for an edge vertex

Interior



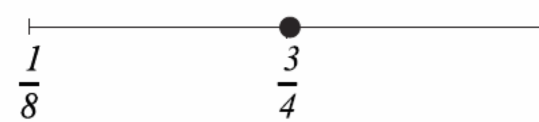
$$\beta = 3/2k; \gamma = 1/4k$$

[Zorin and Schröder,2000]



Mask for a boundary odd vertex

Crease and boundary

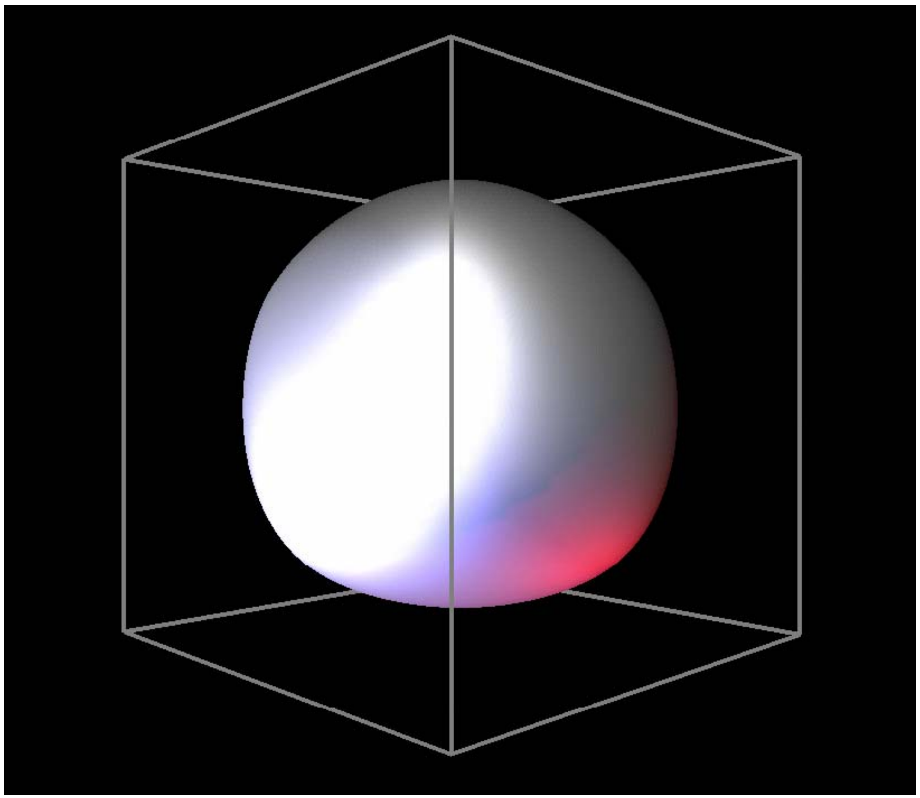


determining subdivision rules

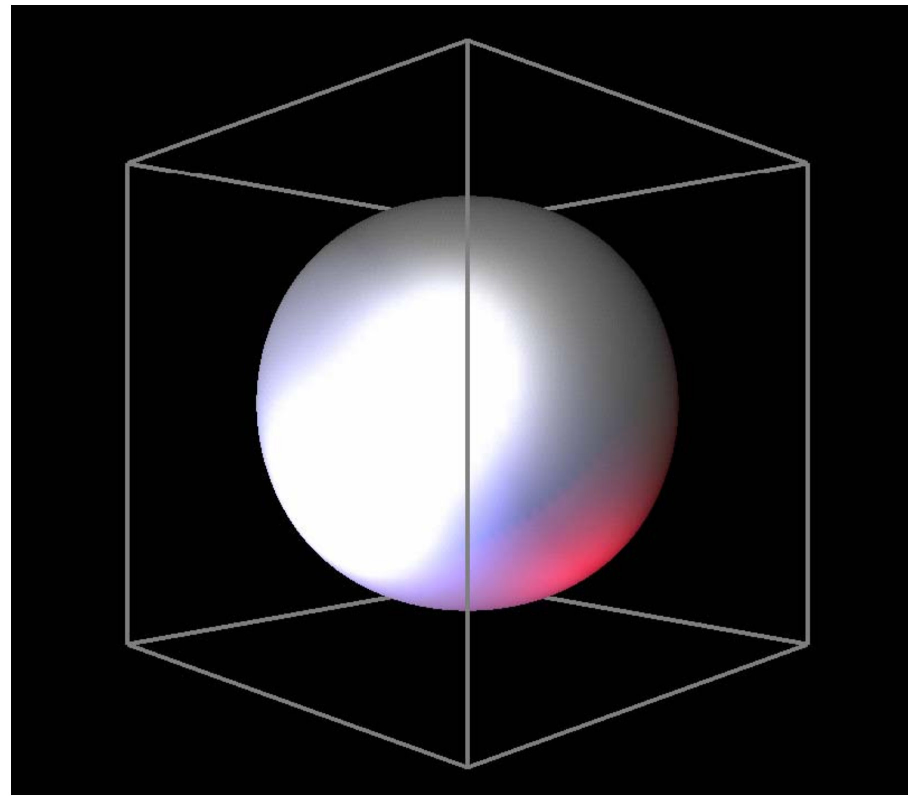
- how to determine subdivision rule?
  - analyze property of limit surface
  - goal: smoothness
- limit surface has provable smoothness!
  - C^1 at extraordinary points
  - C^2 everywhere else

subdivision schemes comparison

Loop



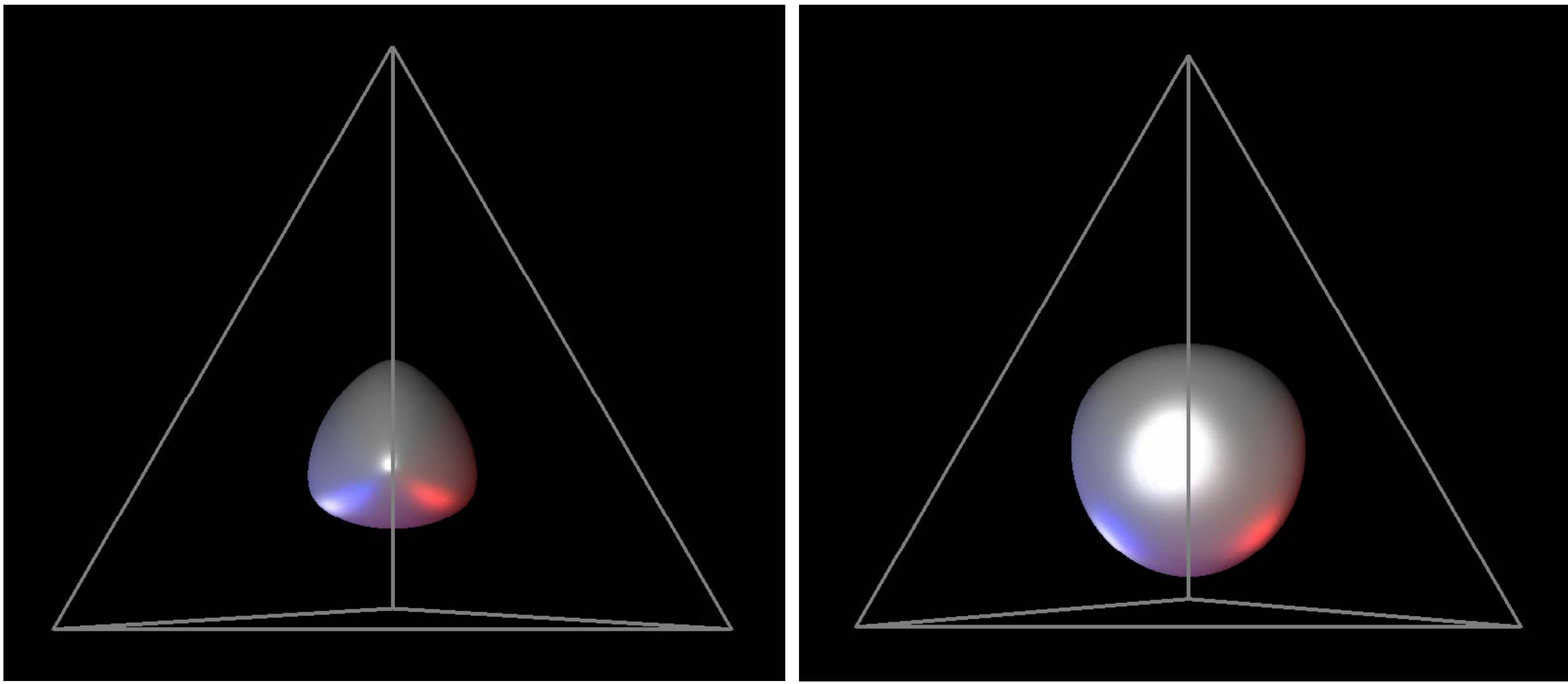
Catmull-Clark



[Zorin and Schröder,2000]

subdivision schemes comparison

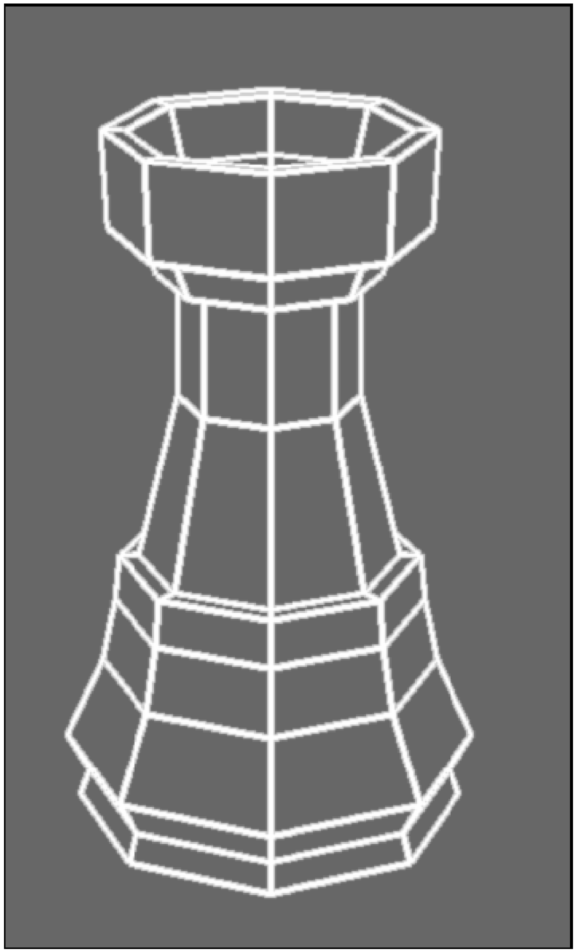
Loop Catmull-Clark



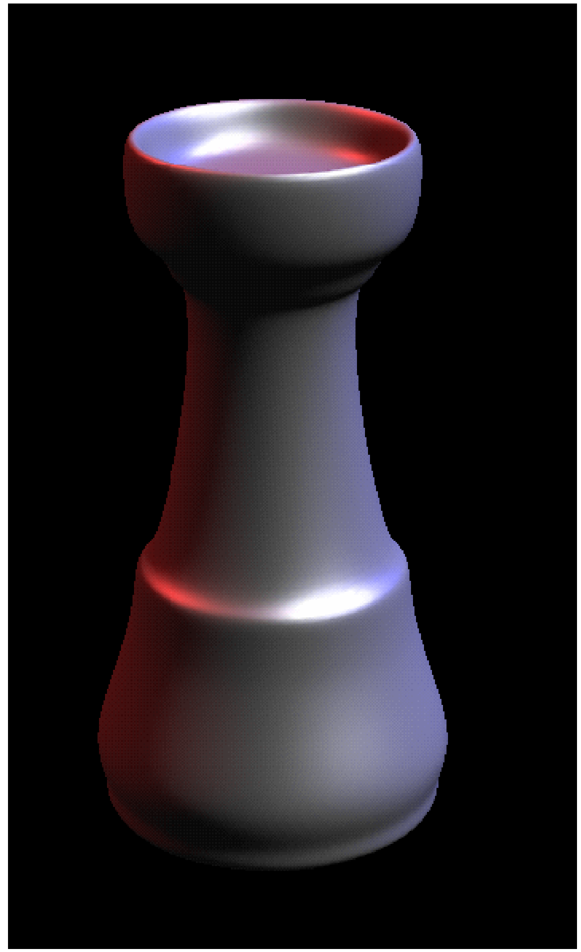
[Zorin and Schröder,2000]

subdivision schemes comparison

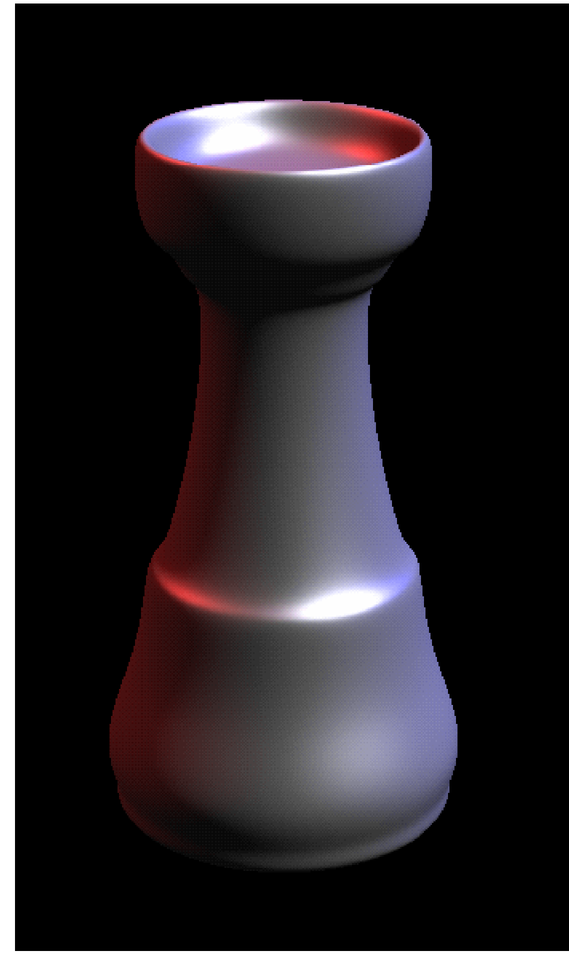
Control Mesh



Loop



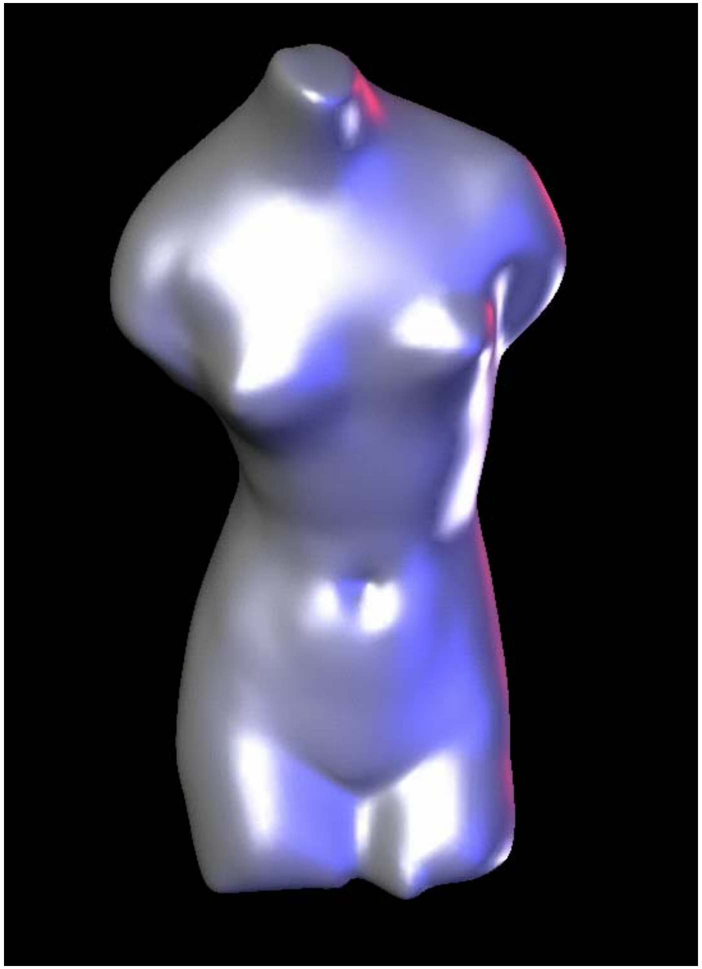
Catmull-Clark



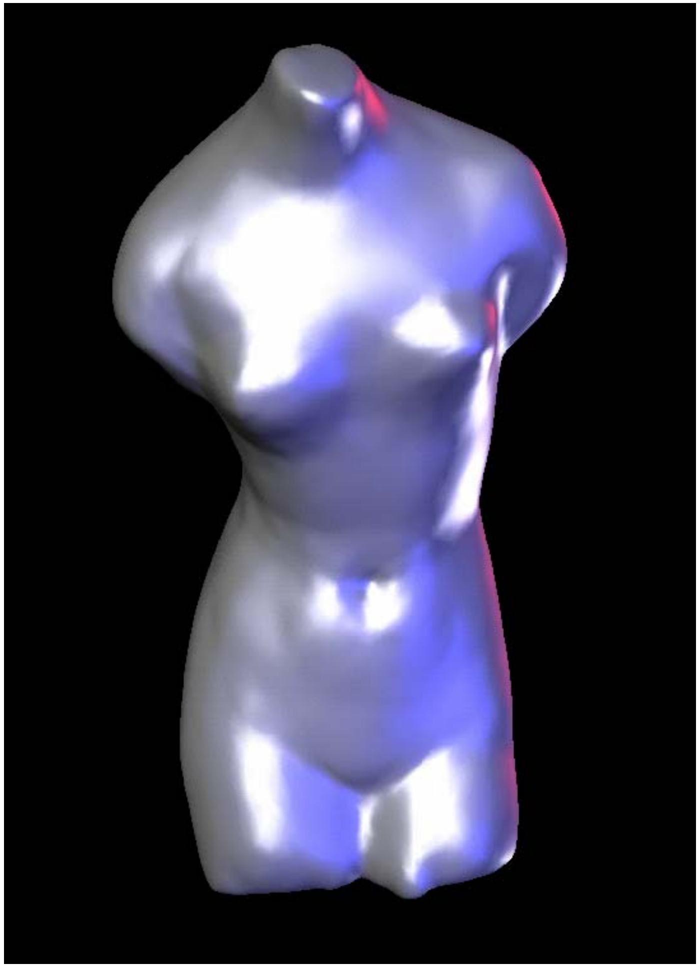
[Zorin and Schröder,2000]

subdivision schemes comparison

Loop



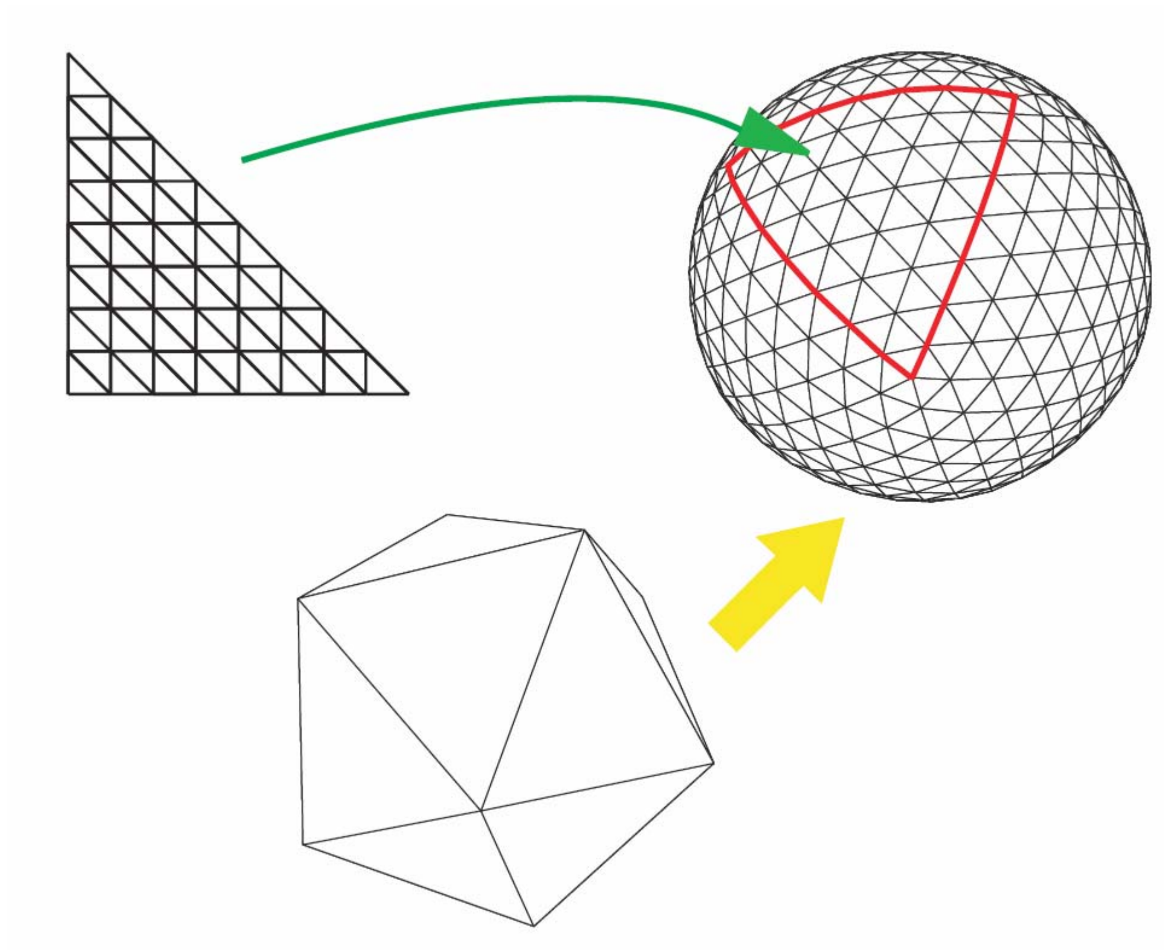
Catmull-Clark



[Zorin and Schröder,2000]

subdivision surface manipulation

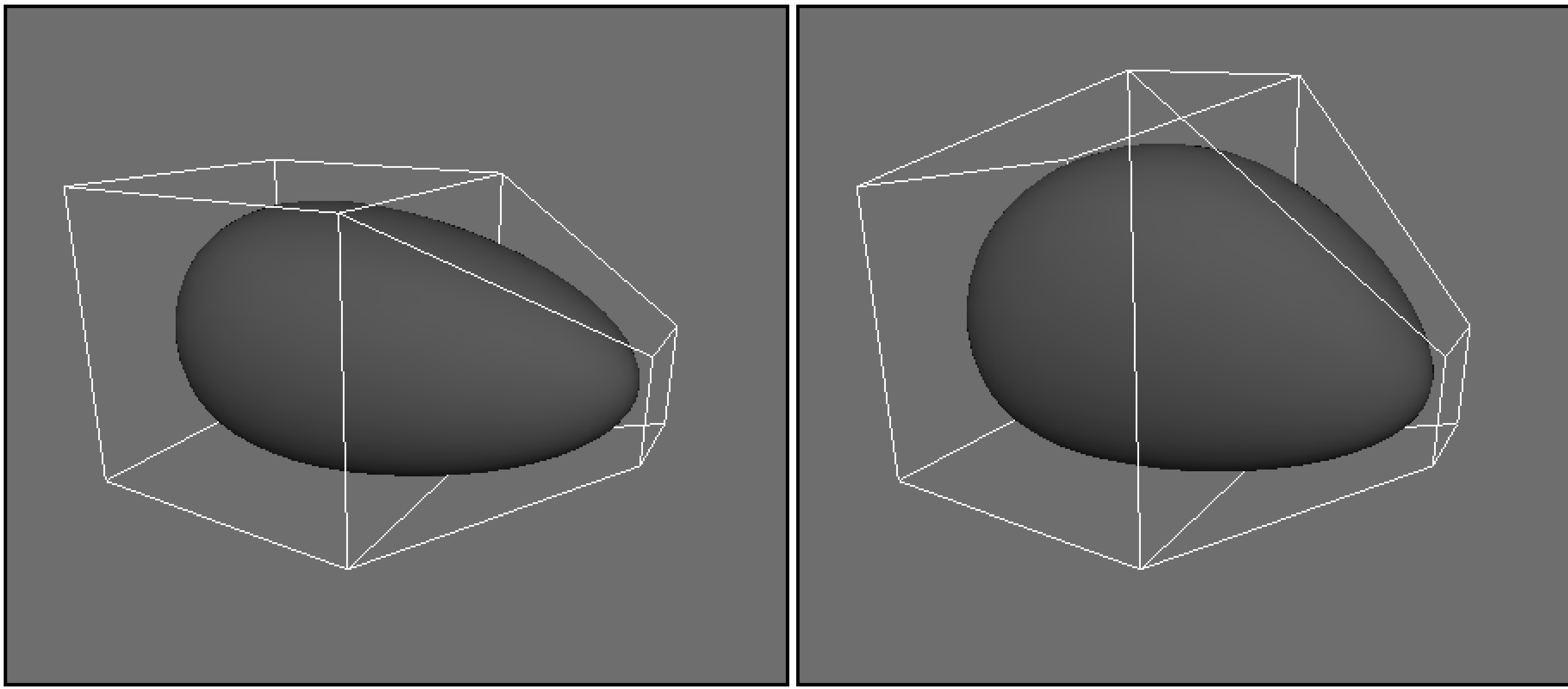
- manipulate original mesh, while displaying subdiv



[Zorin and Schröder,2000]

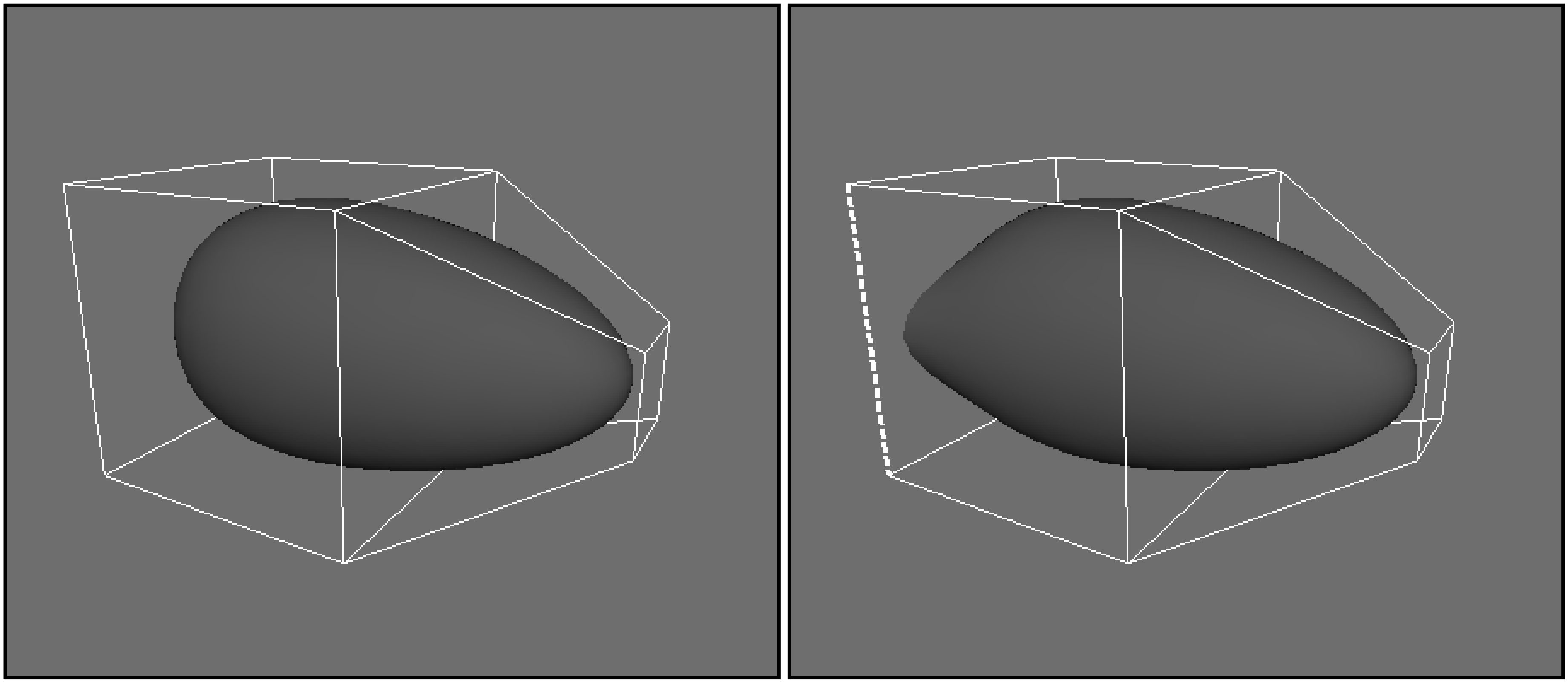
subdivision surface manipulation

- manipulate original mesh, while displaying subdiv



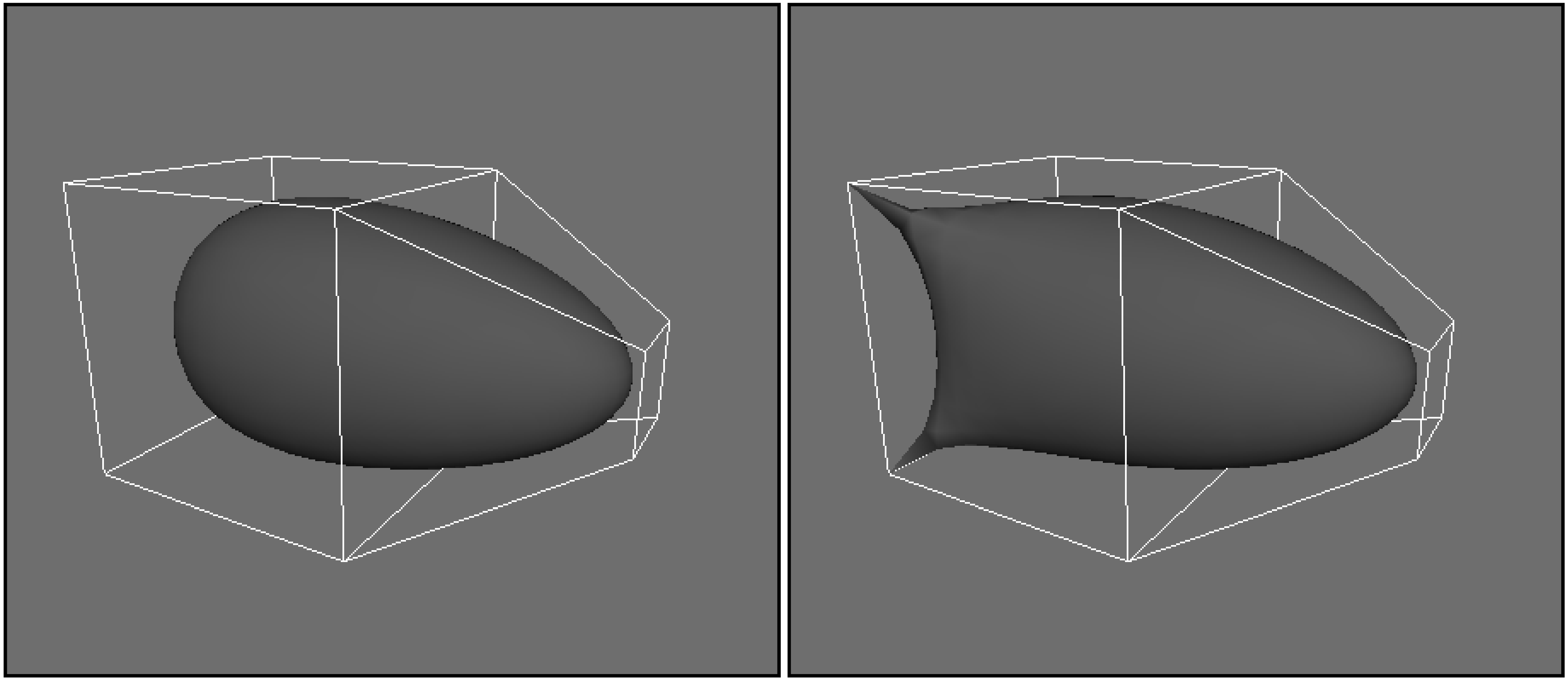
subdivision surface manipulation

- add crease for sharp features
  - on edges



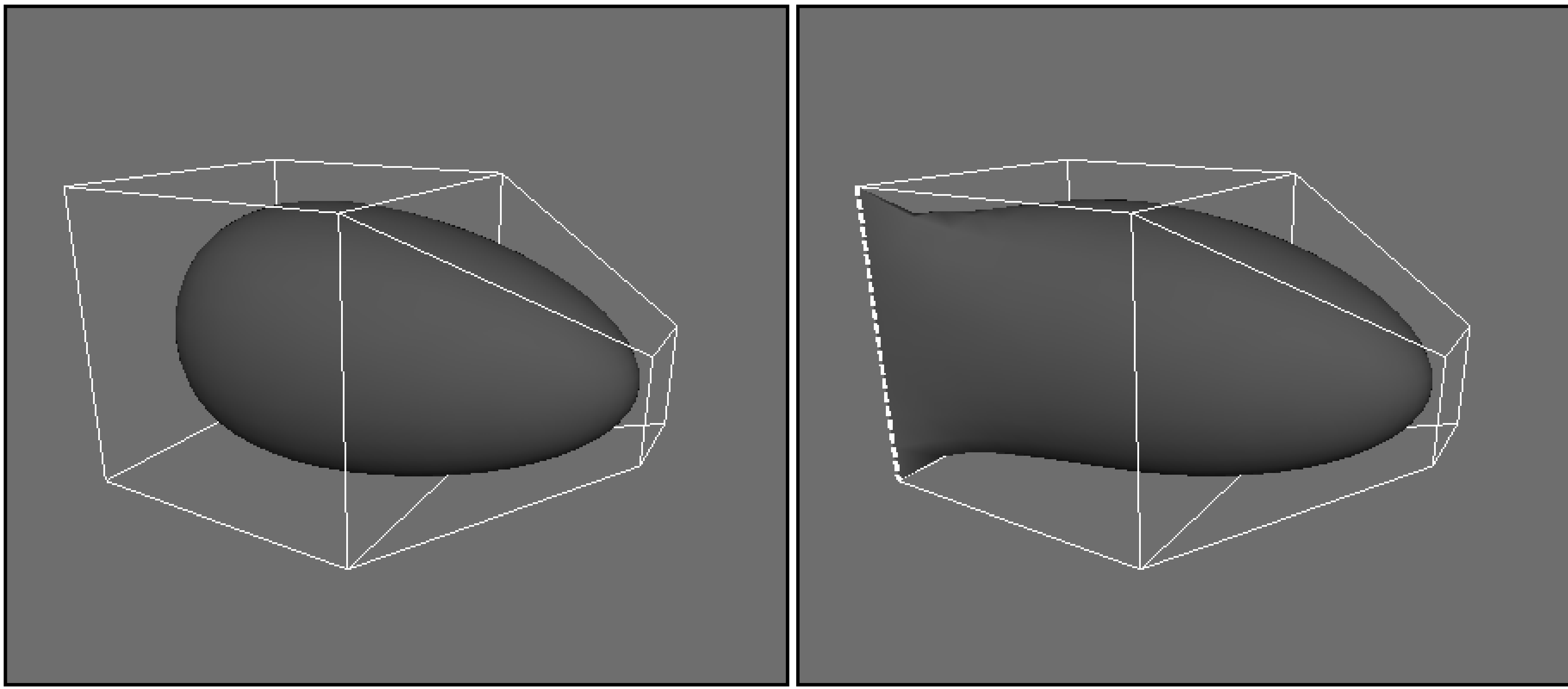
subdivision surface manipulation

- add crease for sharp features
  - on vertices



subdivision surface manipulation

- add crease for sharp features
  - on edges and vertices



subdivision surfaces

- heavily used in Computer Graphics
  - old algorithms but practicalities introduced recently
  - works well with animation
    - arbitrary topology, no stitching
  - Pixar's *Geri's Game* 1998
    - Catmull-Clark subdiv
    - complex control mesh
    - heavy use of creases

surface representation comparison

representation comparison

	polygon meshes	implicit surfaces	parametric surfaces	subdivision surfaces
accurate	no	yes	yes	yes
concise	no	yes	yes	yes
intuitive specification	no	no	yes	no
local support	yes	no	yes	yes
affine invariant	yes	yes	yes	yes
arbitrary topology	yes	no	no	yes
guaranteed continuity	no	yes	yes	yes
parameterization	no	no	yes	no
efficient display	yes	no	yes	yes
efficient intersection	no	yes	no	no

[based on Finkelstein 2004]

