parametric surfaces

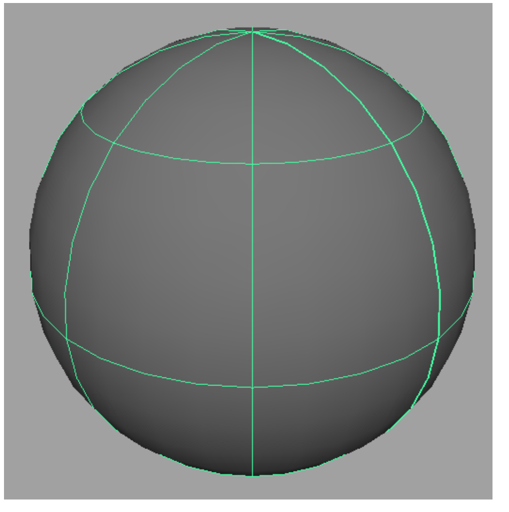
implicit representation for surfaces

- surfaces can be represented implicitly as

$$f(\mathbf{p}) = f(x, y, z) = 0$$

- example: sphere of radius *r* centered at origin

$$x^2 + y^2 + z^2 - r^2 = 0$$



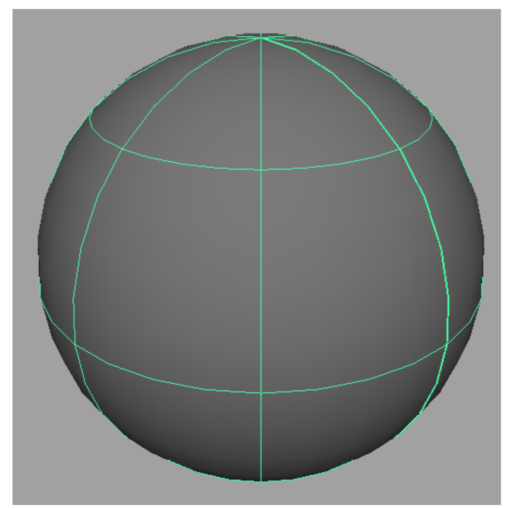
parametric representation for surfaces

- surfaces can be represented parametrically as

$$\mathbf{p}(u,v) = \begin{cases} x = f_x(u,v) \\ y = f_y(u,v) \\ z = f_z(u,v) \end{cases}$$

- example: sphere of radius *r* centered at origin

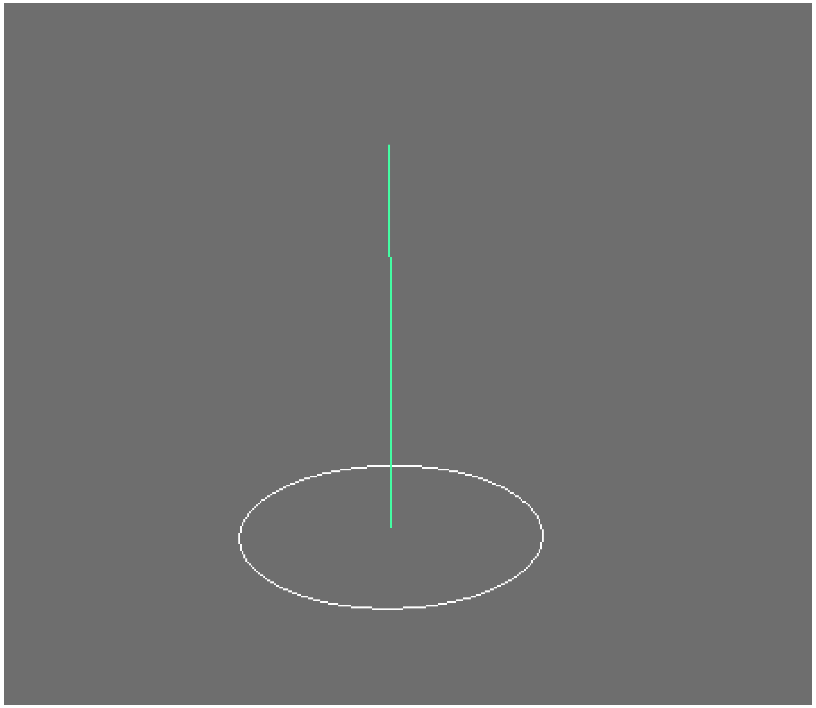
$$\begin{cases} x = r\cos(u)\cos(v) \\ y = r\sin(u)\cos(v) \\ z = r\sin(v) \end{cases}$$

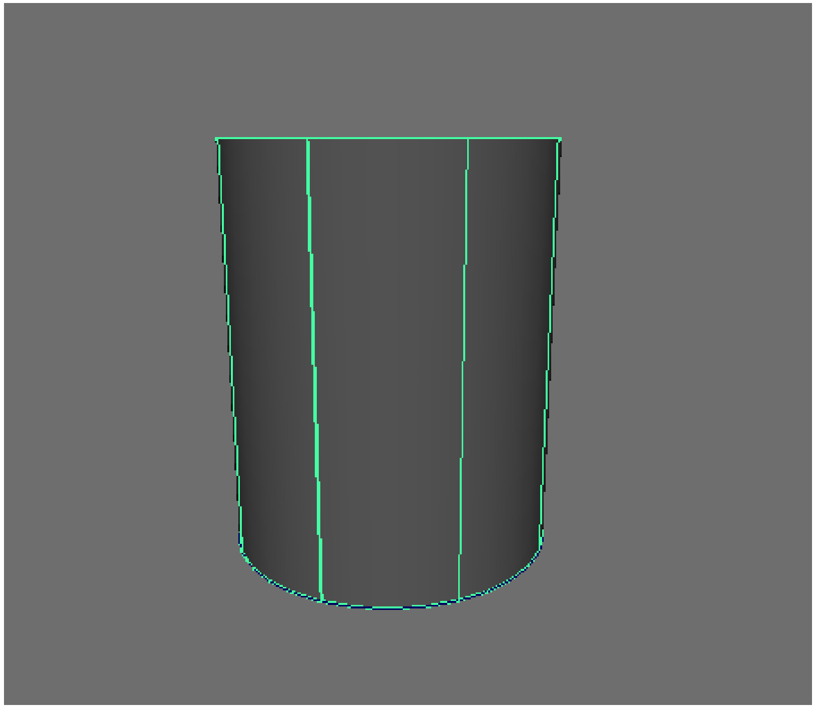


extrusion along an axis

- extrude a 2d curve cross session into a tube
  - for a spline s(u) in xy plane extruded along z

$$\mathbf{p}(u,v) = \left[s_x(u), s_y(u), v\right]^T$$

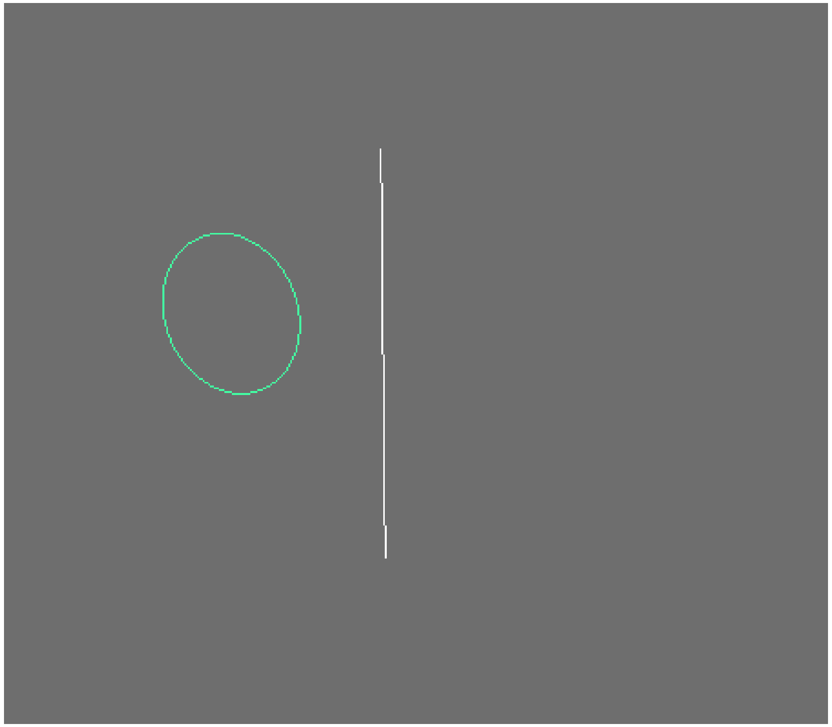


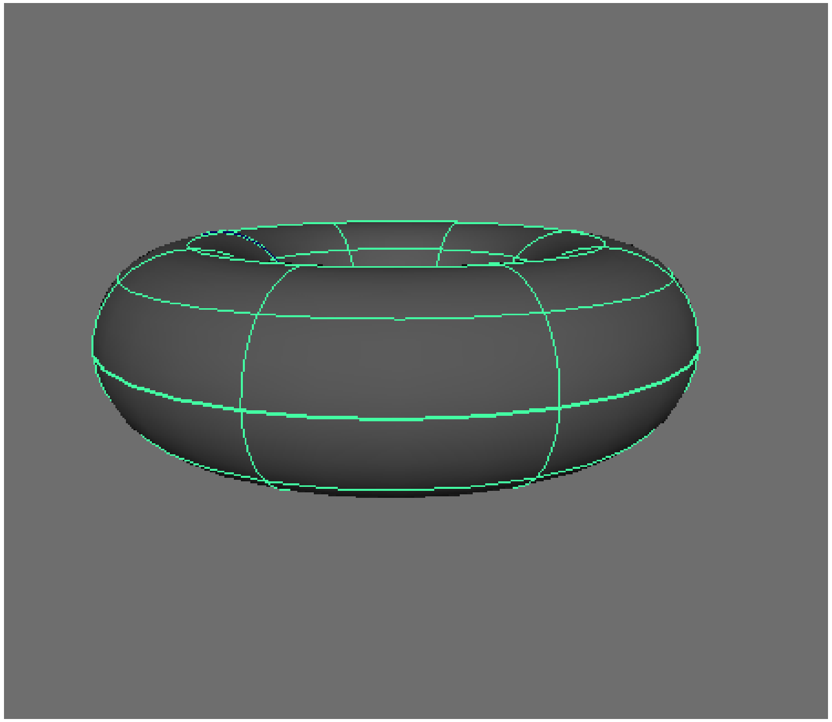


surface of revolution

- spin a 3d curve profile around an axis
  - for a spline s(u) in xz plane revolved around z

$$\mathbf{p}(u,v) = \left[s_x(u)\cos(v), s_x(u)\sin(v), s_y(u)\right]^T$$





swept surfaces

- extrude a 2d curve cross section along a 3d curve
  - more control than extrusion
- draw different splines as cross sections
  - smooth surface

surface patches

- curves: 1d blending functions
- surfaces: 2d blending functions
  - differ in blending functions
  - cross product of 2d blending functions
  - bilinear
  - bicubic Bezier

$$\mathbf{p}(t) = \sum_{i=0}^{3} b_i(t)\mathbf{p}_i$$

$$\mathbf{p}(u,v) = \sum_{i=0}^{15} b_i(u,v)\mathbf{p}_i$$

surface patches

- matrix formulation
  - curves

$$\mathbf{p}(t) = \begin{bmatrix} t^3 & t^2 & t & 1 \end{bmatrix} \begin{bmatrix} m_{11} & m_{12} & m_{13} & m_{14} \\ m_{21} & m_{22} & m_{23} & m_{24} \\ m_{31} & m_{32} & m_{33} & m_{34} \\ m_{41} & m_{42} & m_{43} & m_{44} \end{bmatrix} \begin{bmatrix} \mathbf{p}_0 \\ \mathbf{p}_1 \\ \mathbf{p}_2 \\ \mathbf{p}_3 \end{bmatrix} = \mathbf{t}^T M [\mathbf{p}_i]$$

  - surfaces

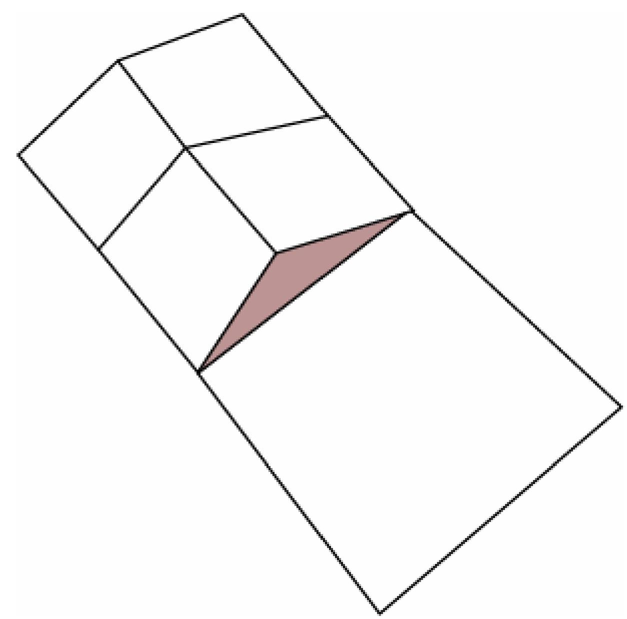
$$\mathbf{p}(u,v) = \mathbf{u}^T M [\mathbf{p}_{ij}] M^T \mathbf{v}$$

surface patches

- joining is hard
  - regular corner: similar to curves
  - irregular corners: hard
- messy to make anything with complex topology
  - only easy for plane, cylinder, sphere topology
  - possible in other cases, just really hard

drawing parametric surfaces

- create triangles mesh
  - similar to curve drawings using lines
- uniform subdivision
  - easy but generates too many triangles
- adaptive subdivision
  - similar to curves
  - check out for cracks



parametric surfaces

- rarely used for complex “organic” object
  - topology limitations too constraining
- used in CAD

