# viewing



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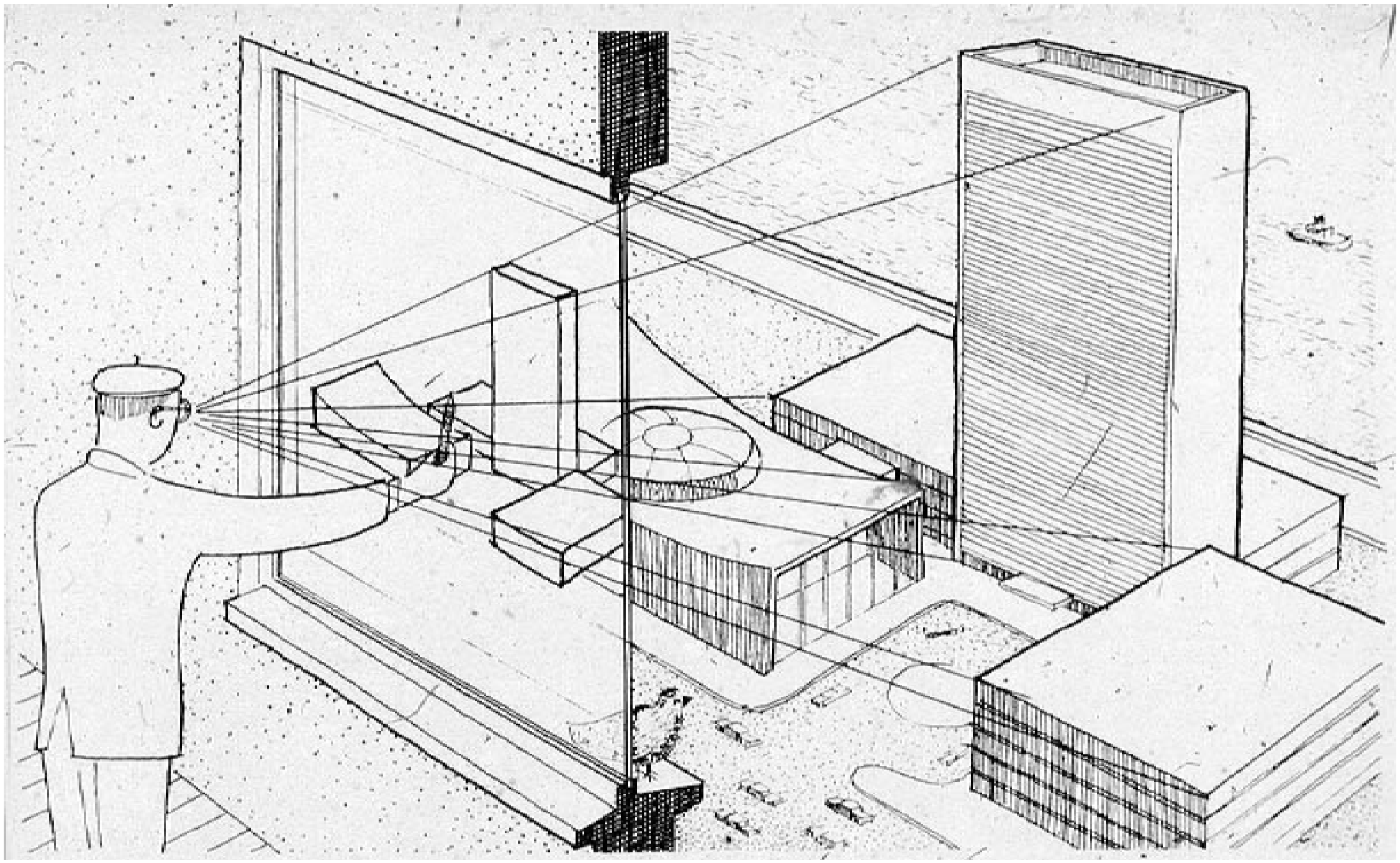
# perspective projection in drawing



[Marschner 2004 – original unkwon]

# perspective projection in drawing

- perspective was not used until circa 15$^{th}$ century
- technical explanation by Leon Battista Alberti
  - 1436, De Pictura – Della Pittura
  - "Trovai adunque io questo modo ottimo cosi in tutte le cose seguendo quanto dissi, ponendo il punto centrico, traendo indi linee alle divisioni della giacente linea del quadrangolo."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# perspective projection in drawing

[1320-1325, Giotto]
[Web Galery of Art, www.wgu.hu]



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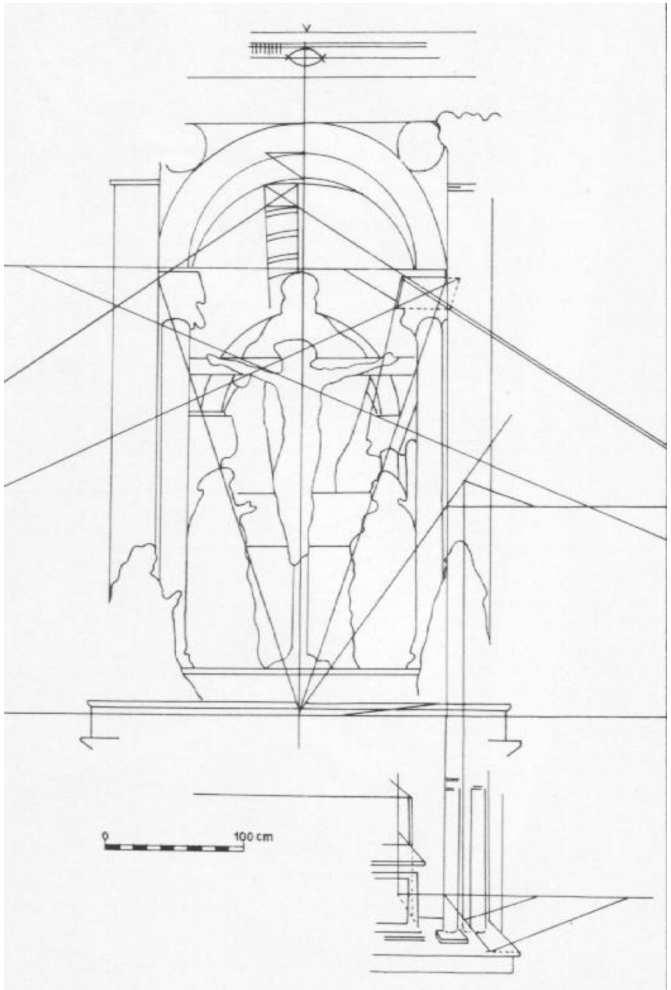
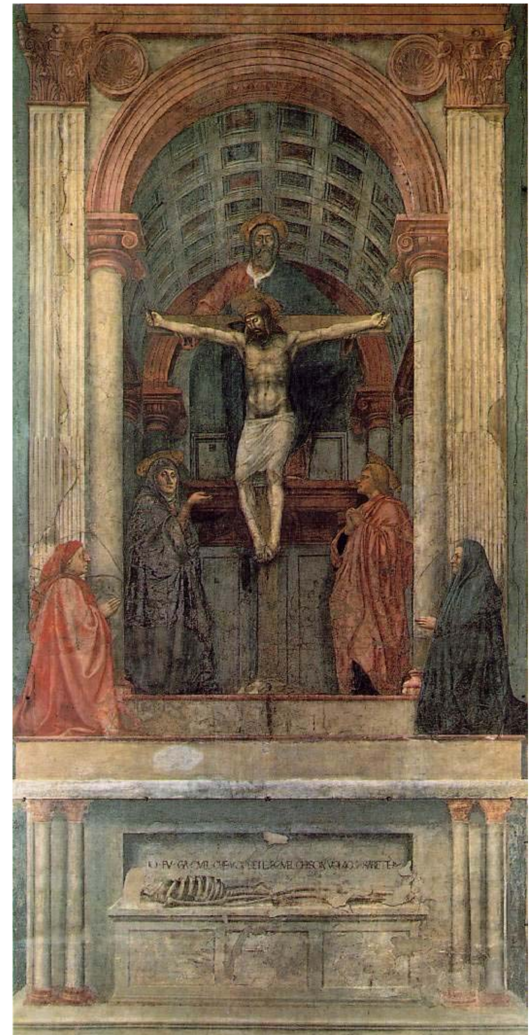
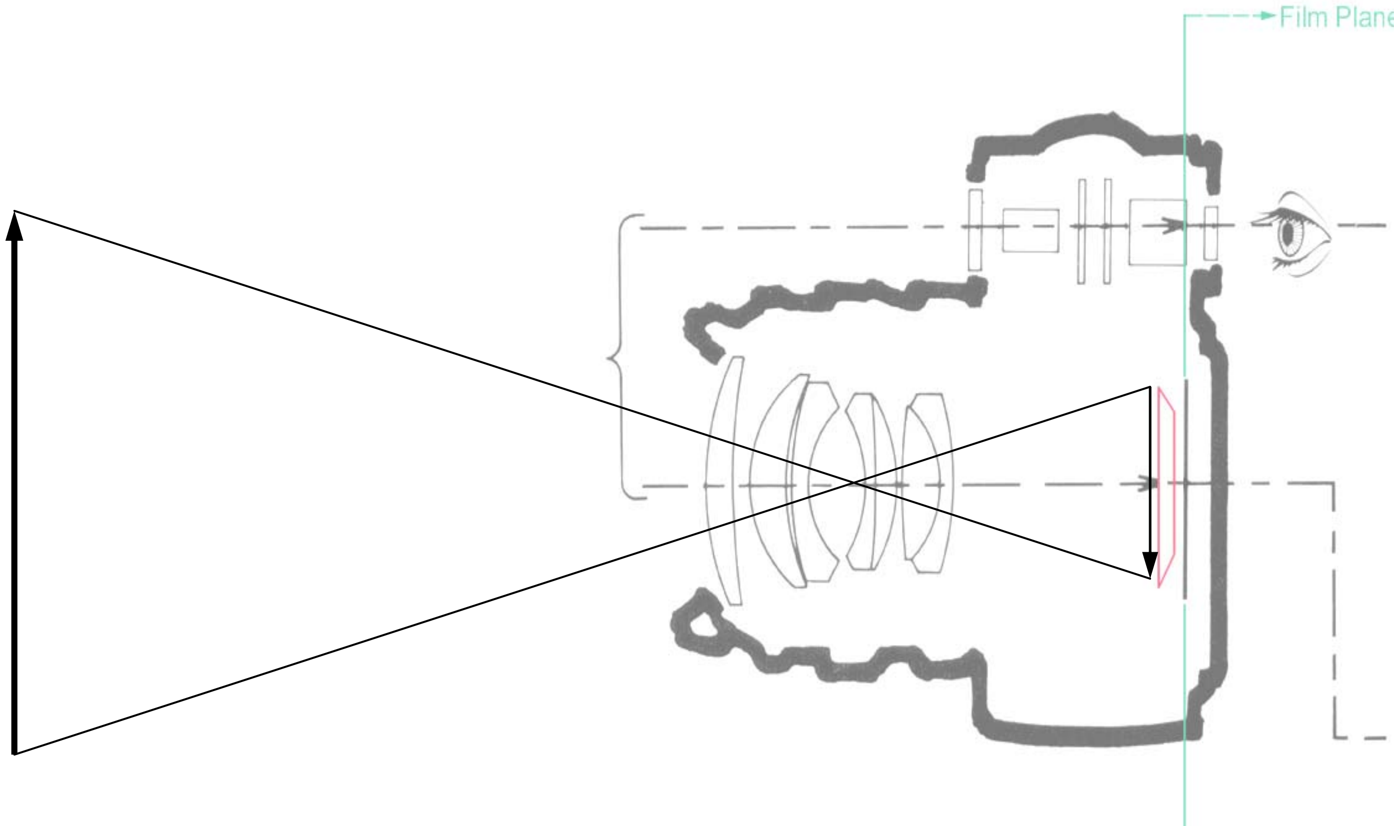
# perspective projection in drawing



[1425-1428, Masaccio]
[Web Galery of Art, www.wgu.hu]

# perspective projection in photography




[Marschner 2004 – original unkwon]

# perspective projection in photography

[Richard Zakia]



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# raytracing vs. projection

- in ray tracing: image plane → object point
  - start with image point
  - generate a ray
  - determine the visible object point
- in projection: object point → image plane
  - start with an object point
  - apply transforms
  - determine the image plane point it projects to
- inverse process

# viewing

- map 3d world points to 2d image plane positions
  - two stages

- viewing transform
  - map world coordinates to camera coordinates
  - change of coordinate system
- projection
  - map camera coordinates to image plane coordinates
  - orthographic or perspective


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# viewing transform

- any affine transform
- useful to define one for our viewer model
  - defined by origin, forward, up
- computed by
  - orthonormalized frame from the vectors
  - construct a matrix for a change of coord. system
  - seen in previous lecture

# projection

- in general, function that transforms points from $m$-space to $n$-space where $m>n$

- in graphics, maps 3d points to 2d image coordinates
  - except we will keep around the third coordinate


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# canonical view volume

- the result of a projection
  - everything projected out of it will not be rendered

- (x,y) are image plane coordinates in [-1,1]x[-1,1]
- keep around the z normalized in [-1,1]
  - define a near and far distance
    - everything on the *near plane* has z=1
    - everything on the *far plane* has z=-1
    - inverted z!
  - will become useful later on


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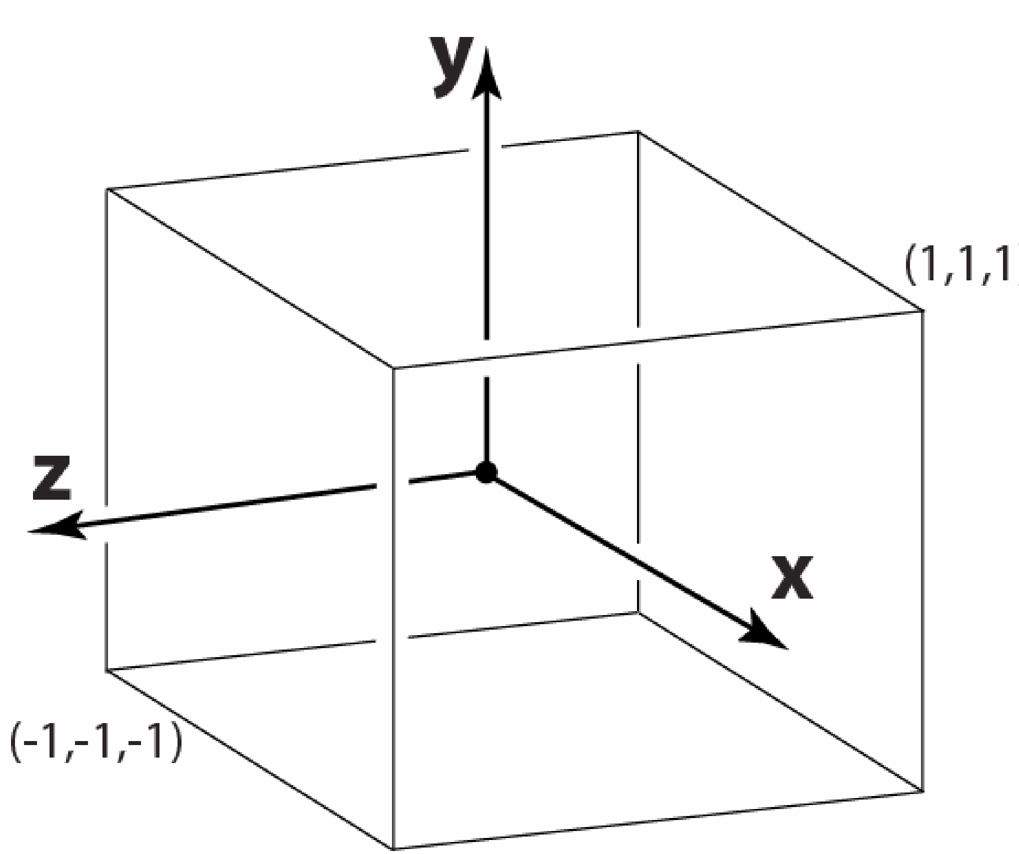
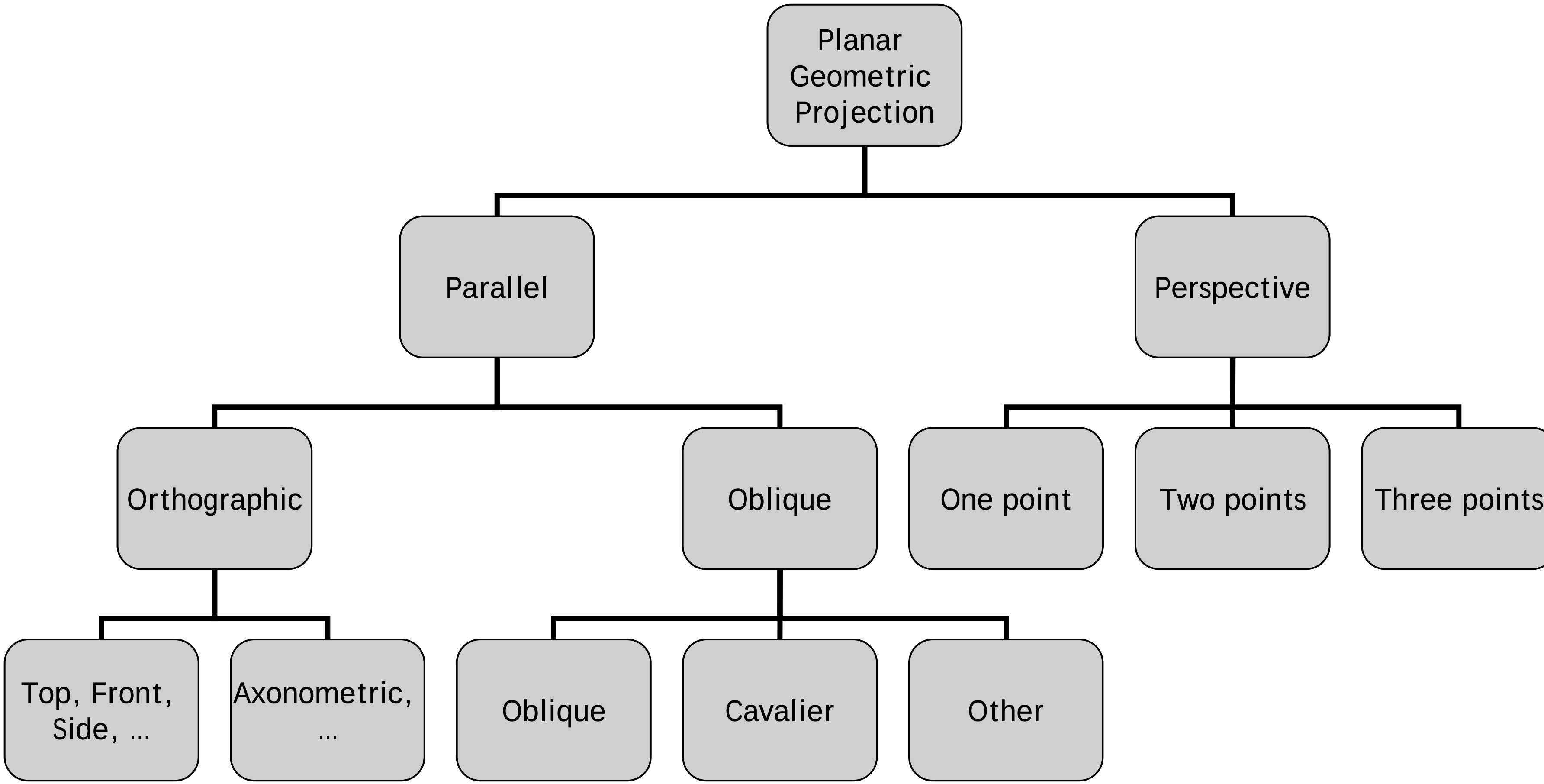
# canonical view volume

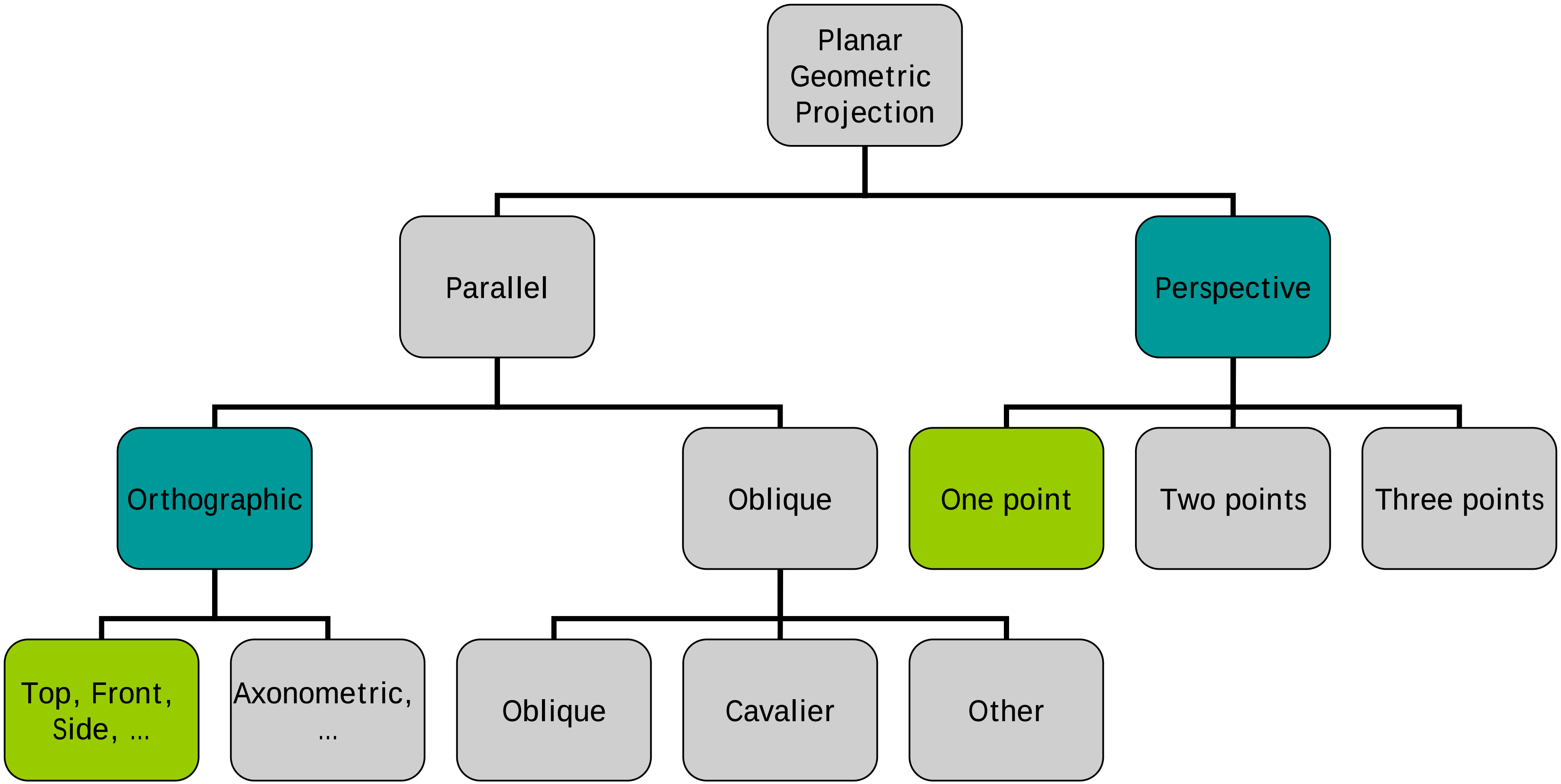
- why introducing near/far clipping planes?
  - mostly to reduce z range, motivated later



# taxonomy of projections

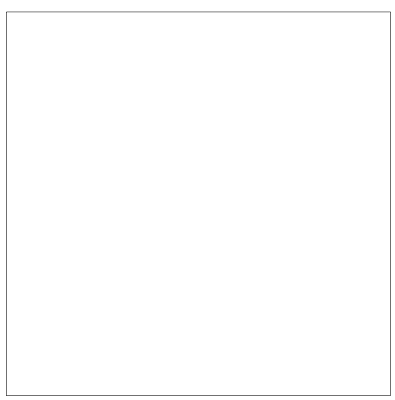


# taxonomy of projections

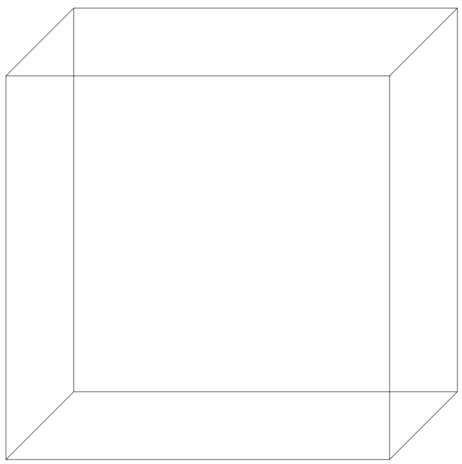


# taxonomy of projections

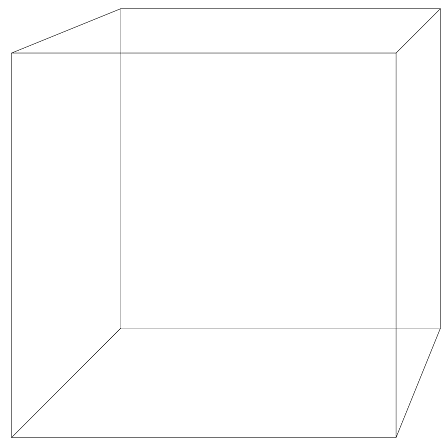
Orthographic



Oblique

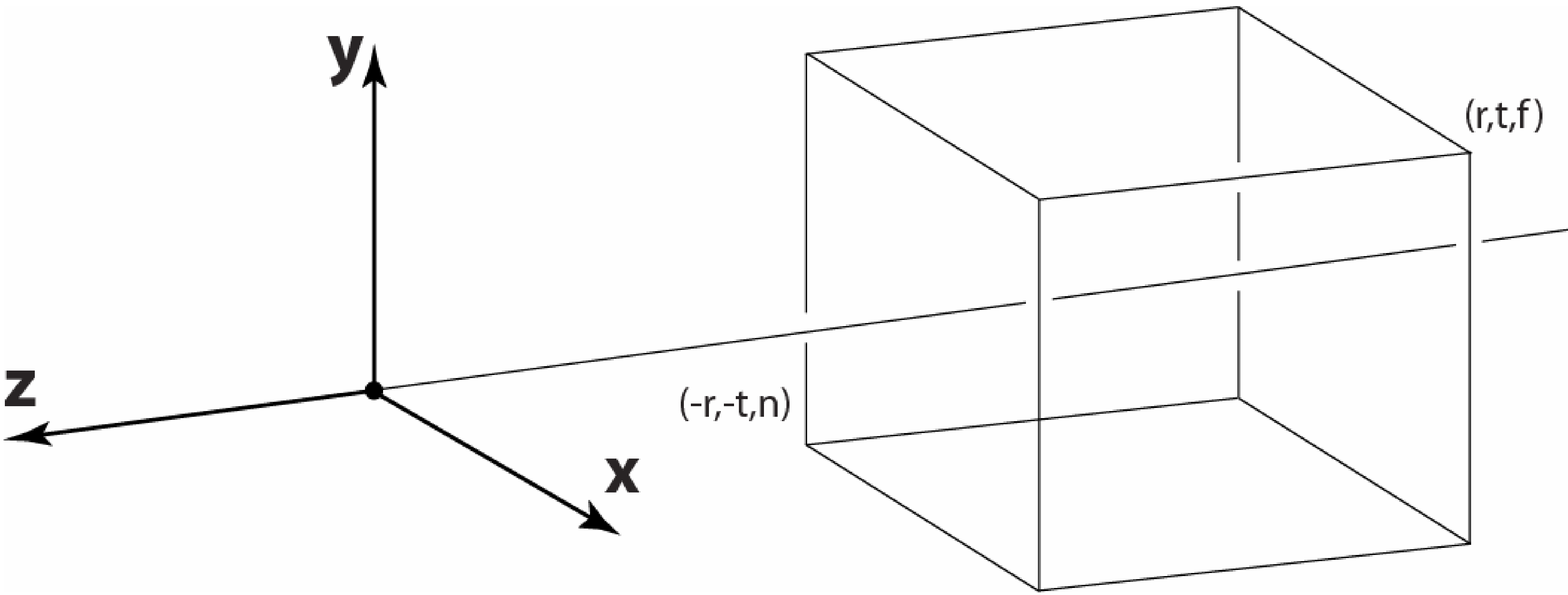


Perspective



# orthographic projection

- box view volume

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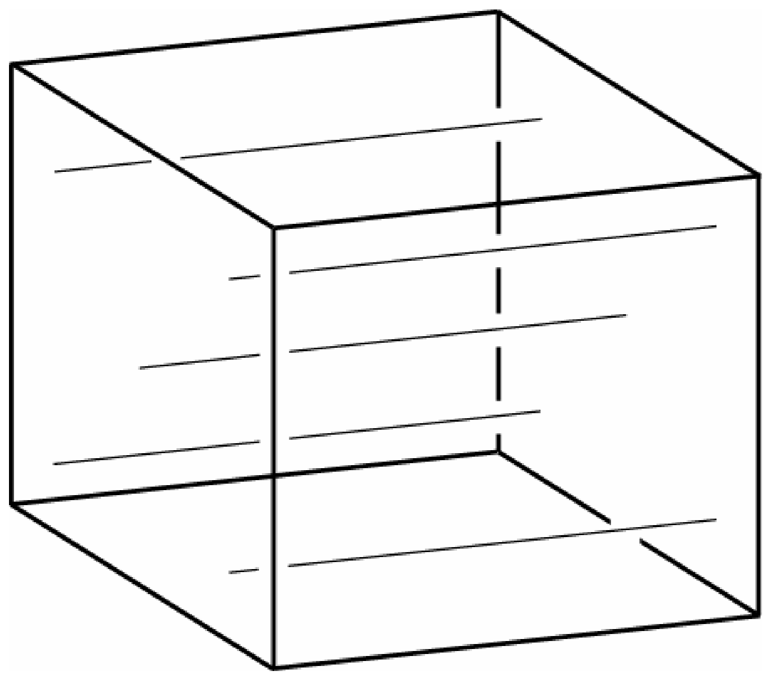
# orthographic projection

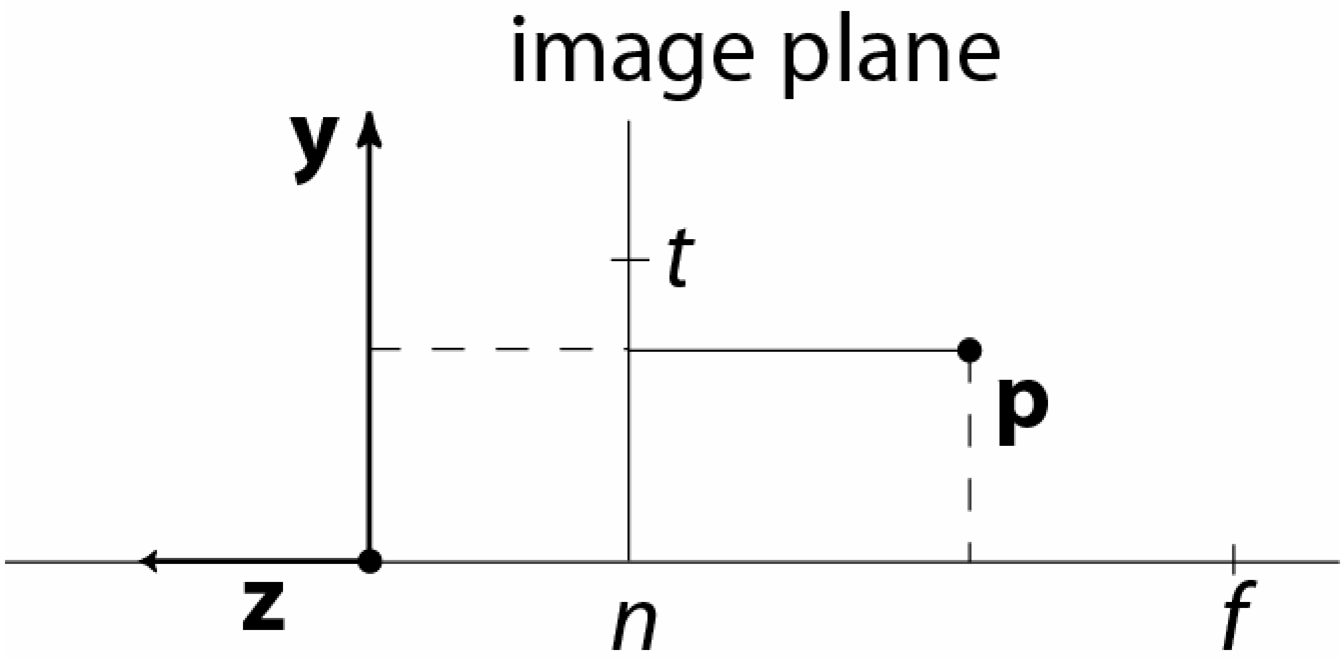
- viewing rays are parallel



# orthographic projection

- centered around z axis

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} x/r \\ y/t \\ (2z-n-f)/(n-f) \end{bmatrix}$$

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# orthographic projection

- write in matrix form

$$\begin{bmatrix} 1/r & 0 & 0 & 0 \\ 0 & 1/t & 0 & 0 \\ 0 & 0 & 2/(n-f) & -(n+f)/(n-f) \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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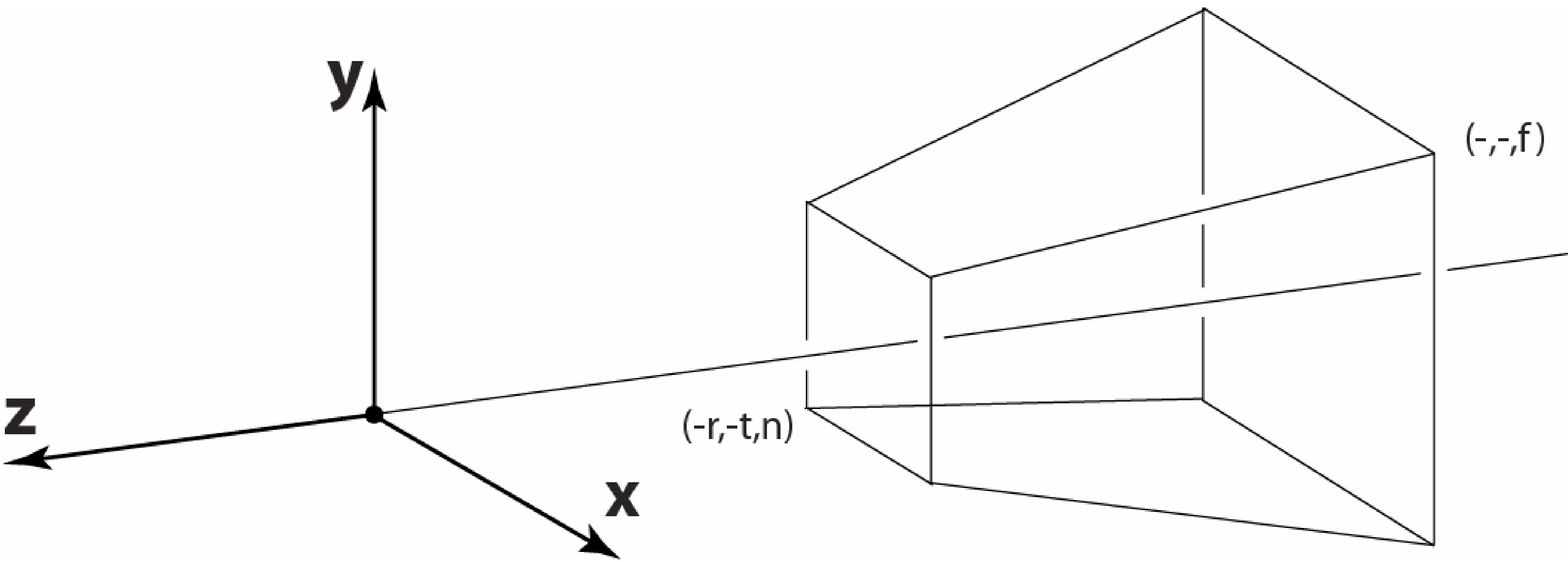
# perspective projection

- truncated pyramid view volume



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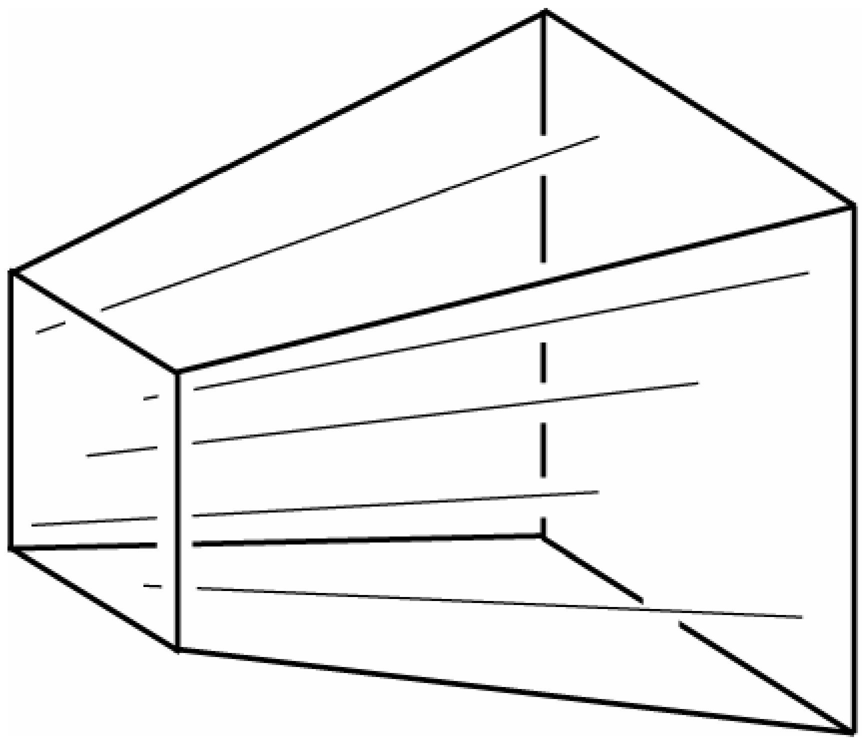
# perspective projection

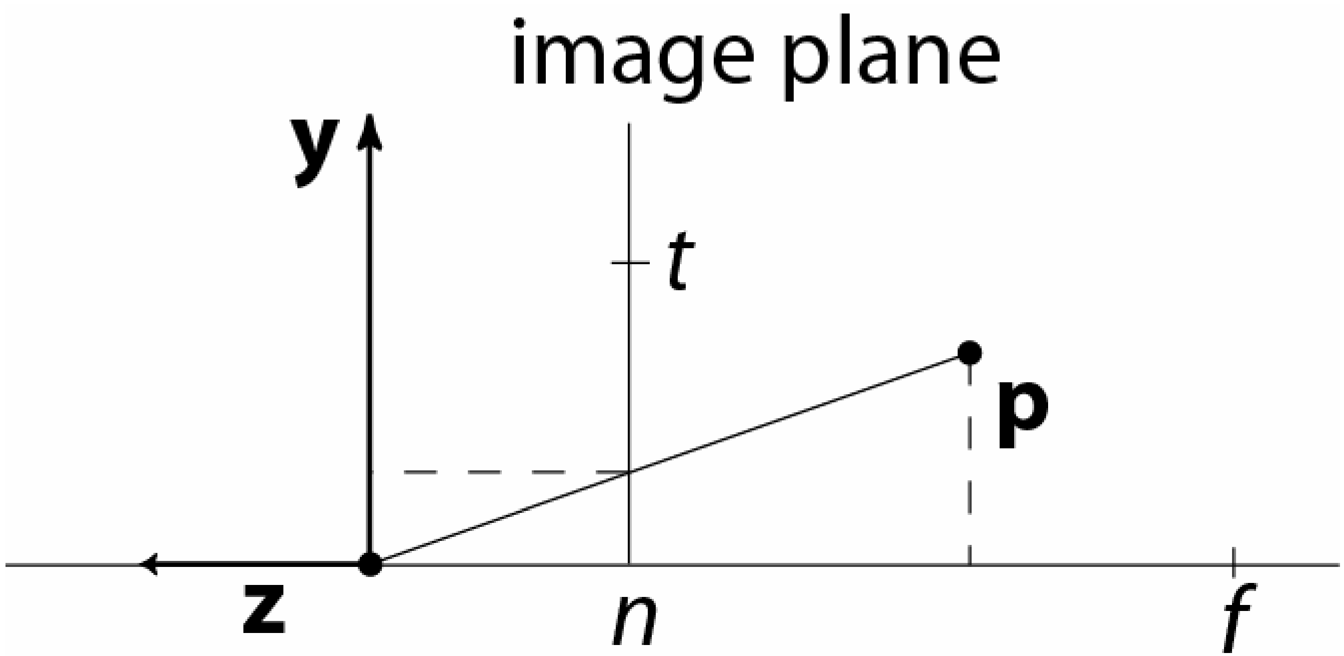
- viewing rays converge to a point



# perspective projection

- centered around z axis

$$\begin{bmatrix} x' \\ y' \\ z' \end{bmatrix} = \begin{bmatrix} (nx)/(rz) \\ (ny)/(tz) \\ \ldots \end{bmatrix}$$



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# perspective projection

- write it in matrix form
  - use homogeneous coordinates, since w≠1!

$$\begin{bmatrix} n/r & 0 & 0 & 0 \\ 0 & n/t & 0 & 0 \\ 0 & 0 & (f+n)/(n-f) & -2fn/(n-f) \\ 0 & 0 & 1 & 0 \end{bmatrix}$$


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# perspective projection

- orthographic projection is affine
- perspective projection is not
  - *does not map origin to origin*
  - maps lines to lines
  - *parallel lines do not remain parallel*
  - *length ratios are not preserved*
  - closed under composition



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# more on projections

- the given matrices are simplified cases
- should be able to define more general cases
  - non-centered windows
  - non-square windows
- can find derivation in the Shirley's book
  - but it is a simple extension of these

- note that systems have different conventions
  - pay attention at their definition
  - sometimes names are the same

# general orthographic projection

$$\begin{bmatrix} \frac{2}{r-l} & 0 & 0 & \frac{l+r}{l-r} \\ 0 & \frac{2}{t-b} & 0 & \frac{b+t}{b-t} \\ 0 & 0 & \frac{2}{n-f} & \frac{n+f}{n-f} \\ 0 & 0 & 0 & 1 \end{bmatrix}$$



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# general perspective projection

$$\begin{bmatrix} \frac{2n}{r-l} & 0 & \frac{l+r}{l-r} & 0 \\ 0 & \frac{2n}{t-b} & \frac{b+t}{b-t} & 0 \\ 0 & 0 & \frac{f+n}{n-f} & \frac{2fn}{f-n} \\ 0 & 0 & 1 & 0 \end{bmatrix}$$


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